Vol.22 No.1

Egnals

Realising potential in mathematics for all

for ages 3 to 18+ ISSN 1465-1254

Fun and games in the mathematics classroom

Equals

Realising potential in mathematics for all

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(All publishing and subscription enquiries to be addressed here.)

Designed by Nicole Lane

Corrections and clarifications:

Objects to think with. Papert (1980) (*Equals 21.2*, page 19)
This was not written by Mary Clark but by Pete Jarret. We apologise for any misunderstanding this may have caused.

Editor's Page

I think we should have called this edition 'The Joy of Maths'. This is a timely message for all of us, no matter who we teach. My own daughter is an excellent mathematician who got real joy from her studies up to GCSE level. At A level the situation was very different and she ended up dropping the subject at the end of Year 12 with the claim that she was 'Not very good at maths!' This is clearly not true as she gained 90% in her AS level.

Several articles in this edition of *Equals* suggest a range of ways that the joy of our subject can be promoted and sustained. It is however too late for one of my children.

In this vein Pete Jarrett suggests seven ways that enjoyment can be promoted within any mathematics classroom. Mark Pepper takes a different angle and provides a historical context for anyone who wishes to create an environment that is both enjoyable and rewarding. Mary Clark feels that games are a good way forward for all learners and shares two that she feels are particularly productive.

The latter half of this edition sees Mark Pepper outline an approach to mathematics that will engage the visually impaired. Maths Talk comes from a conversation, on graphs, between three Year 9 students.

Please respond to Mary Clark who uses this edition to re-launch the Harry Hewitt Memorial Award. I had the great honor of visiting a school to present this award several years ago. The aim of the prize is to celebrate the success of those who are rarely acknowledged. Please read her piece and reflect upon which of your pupils have made a breakthrough which has given you reason to celebrate and do nominate them.

Before Christmas I was listening to my nephew talking about his mathematics lessons and I thought he is not having fun! I spent the holidays reflecting upon this and the idea stayed with me and I was pondering it during a classroom support visit. My own view is that if we are able to engage our learners in the task at hand then enjoyment and fun will emerge as a consequence. The two teachers I was working with had decided to team-teach a lesson and asked me to look for ways that they could increase the engagement of their students.

At the start of the observation I wrote in my note pad:

- Engagement does not = giving pupils fun things to do.
- Engagement does = making the challenge accessible and activating the pupils to think and talk.

I don't know if Ben and Jerry (not their real names) knew what they were doing and just how engaged their pupils were. The class were low ability but right from the start they had the opportunity to make sense, at their own level, of the challenge set. Once they were ready to talk Ben and Gerry carefully

used the emerging ideas to draw the pupils further into their learning intentions. It was lovely at the end to be able to share with them just what I had seen and also for them to realise just how engaged their students were. Yes some pupils were off task but the overwhelming feeling was that throughout the lesson the pupils were keen to share their ideas and were able to talk right to the bell.

In December I met with Barbara Rodgers, of the Solent MathsHub, and her colleague Julia Brown. They both feel very strongly that there is a real need for support for those who are struggling to teach the least able and see *Equals* as an ideal vehicle for networking and the sharing of ideas. Future editions will aim to be as practical as possible, including sample activities and ideas for you to try.

There will be an *Equals* session at the Easter Conference. Please come along and say hello and share what you are doing, and what help you would like, to meet the needs of your learners.

I will end with another appeal – please get in touch to share the things you have been doing in your classroom to support the learning of your pupils. All hints, tips and ideas will be gratefully accepted and shared.

7 things you can do to help create a happier maths classroom

In this piece Pete Jarret asks some very tough questions and outlines seven rules of thumb that can be used to create a classroom where fun and learning go hand in hand. There is much wisdom in this article and I am sure it will be followed up during the *Equals* session at the 2017 Easter Conference.

1. Know your learners

Get to know how they think about maths, how they learn, what they find easy and what they find difficult. Numeracy is an important tool in modern life and the modern workplace, but being adequately numerate requires a concept of numbers, let's call this numbersense, a knowledge of learning, the ability to think mathematically, and the skill of wading through a whole raft of cultural misconceptions, memes and expectations without losing all sense of confidence and efficacy.

Assess your learner's basic numeracy skills, their comfort with the rules of arithmetic, and note any misconceptions that need acting on immediately. No doubt you will all have learnt not to expect complete mastery as learners move to whatever level you have been working on, so get to grips with their errors and misunderstandings.

It's just as important to know how your learners feel about mathematics and learning in general. Do they have a growth mindset? Do they have a particular mathematical learning style? Do they see errors as disasters or places to grow from? Do they have support at home? Are they anxious about maths? How do they do in other subjects?

Professor Steve Chinn's excellent book '*More Trouble with Maths*' (NASEN, 2016) has a number of useful resources that will help you to get to know your learners.

The 'Myself as a Learner Scale' (*Imaginative Minds*, 2014) by the late Professor Robert Burden is a fantastic tool for analysing self-perception of learning and can easily be administered in a group really quickly.

2. Be empathetic and sympathetic

Now you are aware of the learning strengths and difficulties of your learners, act on this to provide a learning environment that avoids creating unnecessary discomfort. In 'The Assessment of Self-efficacy' (*Equals* vol. 20.1) I introduced some research that identified aspects of the maths classroom that students lacked the efficacy to operate successfully in, things like explaining a solution to a problem in front of the group and having to answer quickly. Whilst both attributes are somewhat useful, they are also somewhat stressful. There needs to be a balance between understanding the learning value of challenging situations and the impact on a student's engagement and enjoyment in learning.

Every learner is different, and, whilst it is nigh on impossible to accommodate every individual difference, we should have great regard to the types of difference and the impact this may have on their ability to do mathematics. To give an example, people process information at different speeds, but this does not automatically impair their mathematical ability. Indeed, from my own experience, some of the most accomplished mathematicians are very slow at processing information. With this in mind – how important is the 'need for speed'? Surely we are after efficient mathematicians who work with confidence and accuracy – highlighting speed of processing issues doesn't help anyone and can create serious anxiety, even amongst the most capable.

It is also worth remembering the frighteningly common cultural meme – "I was doing OK at Maths and then I had a teacher that hated me". The thing is, as a bunch of people, maths teachers don't portray an all pervading sense of doom for their learners, and, whilst we may get frustrated with some, shouldn't really dislike anyone. The problem is this, our learners are young and trying to find their way in the world – they are sensitive souls – and therefore, what we consider to be gentle encouragement to engage is seen as something different for learners who find the subject to be difficult and teachers to be lacking in empathy. If we remember this, we may be able to head off such disillusionment before it becomes disengagement.

"I was doing OK at Maths and then I had a teacher that hated me"

Read the case studies on Steve Chinn's '*The Fear of Maths. Sum Hope 3*' (2012, Souvenir Press) and '*It Just Doesn't Add Up. Explaining Dyscalculia and Overcoming Number Problems for Children and Adults*' by Paul Moorcraft (2015, Tarquin Publications) for an insider's viewpoint.

3. Use a range of approaches to learning.

In particular, consider the steps that are gone through when we learn new maths ideas. Bryant, et al. (2015) identify the 'concrete – semi-concrete – abstract' (CSA) teaching sequence, whilst Professor Mahesh C. Sharma describes the six levels of knowledge required to understand a new mathematical idea – intuitive; concrete; pictorial; abstract; application; and, communication.

At its simplest level, mastery is being able to undertake a task with fluency and understanding

The central theme running through both sequences is the movement from concrete manipulatives – for example, Dienes Blocks, Cuisenaire Rods and fraction tiles – through semi-concrete, pictorial, representations to abstract methods which use symbols.

Sharma further develops this by suggesting that the ideas learnt need to be applied in a range of situations, and then reinforced by communicating the idea to others, be that the group, an individual or the teacher.

Approaches that allow for a visualisation of how numbers work are used throughout the NCETM Calculation Guidance for Primary Schools (2015) and are common in Singapore Maths.

The NCETM Guidance is available here: https://www.ncetm.org.uk/public/files/25120980/NCETM+Calculation+Guidance+October+2015.pdf

Read about Professor Sharma's 'Six Levels of Knowledge' Model in Judy Hornigold's '*Dyscalculia Pocketbook*' (2015, Teacher Pocketbooks series).

The chapter 'Learning Disabilities. Mathematics Characteristics and Instructional Exemplars' by Diane Pedrotty Bryant, Bryan Bryant, Mikyung Shin and Kathleen Hughes Pfannenstiel is taken from '*The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties*' edited by Steve Chinn (2015, Routledge). This book contains a number of other useful chapters on evidence based classroom practice.

4. Mastery

Professor Sharma's 'Six Levels of Knowledge' model, above, begins its sequence with intuition. Each new fact is linked to something the learner already knows. Ideally, this is linked to something that the learner can do with an element of automaticity. At its simplest level, mastery is being able to undertake a task with fluency and understanding, so that it can be completed with little cognitive load. How often do we feel that our learners have achieved mastery before we move on to the next topic?

Salman Kahn, the founder of the Kahn Academy, makes an intriguing analogy in his TED Talk 'Let's teach for mastery – not tests scores' when talking about knowledge gaps that reappear later in the curriculum. Imagine building a house and saying you have two weeks to build the foundations, and then something happens, it rains for a week, or the apprentice mixes the concrete wrong, so that the foundations are only 70% complete – you wouldn't just move on to building the first floor, and you certainly wouldn't get the electricians in before you have completed the roof. Unfortunately, we often feel compelled to move on before we have built the solid foundations, which will only lead to struggling learners in the future.

(https://www.ted.com/talks/sal_khan_let_s_teach_for_mastery_not_test_scores?language=en)

5. Discussion – at every opportunity

There are three discussions that should be encouraged in the classroom; discussion for learning, where learners are engaged on whole-class and group tasks together and discuss their approach to problem solving, address gaps in knowledge and create a collaborative and supportive environment. Jennie Pennant provides a useful resource on the NRICH website to support this (https://nrich.maths.org/10341); Discussion as feedback, where the teacher is able to provide constructive and challenging support to help learners develop understanding. This model may draw on the master and apprentice relationship, where the learner is guided towards their own mastery through discussion and a feeling of belonging to a community of practice. '*Situated Learning*' by Jean Lave and Etienne Wenger (1991, Cambridge University Press) is a benchmark text. In many ways, if we are empathetic teachers, we already employ this approach; And, finally, discussion for metacognitive reflection, where we encourage our learners to adopt a growth mindset and view errors as learning opportunities, or just to think 'if I did that wrong this time, what do I need to do to avoid making the same mistake'. Read John Hattie and Gregory Yates' '*Visible Learning and the Science of How We Learn*' for some evidence led analysis.

you will create a positive environment that is disciplined rather than an unruly one requiring disciplinary action

6. Have Clearly defined classroom rules that are on display and are adhered to.

Everybody loves it when they know where they stand. If both yourself and the learners know what is expected in the classroom, you will create a positive environment that is disciplined rather than an unruly one requiring disciplinary action – although we have all had that wet Friday in February!

If we create a positive environment in which people believe they can learn without ridicule, where failure is renamed practice, and where everyone is treated with respect and have clear rules that are targeted at creating this environment, then we all begin to expect learning to happen. '*The Essence of Maths Teaching for Mastery*' from the NCETM sets out some clear expectations that should be

made available to learners. '*Engaging Learners*' by Andy Griffith and Mark Burns (2012, Osiris) has an excellent chapter on extrinsic and intrinsic motivation.

(https://www.ncetm.org.uk/files/37086535/The+Essence+of+Maths+Teaching+for+Mastery+june+2016.pdf)

7. Be fun and energetic and passionate.

Because you love mathematics, it is the most incredible subject, it surrounds us in our everyday lives and you want everyone to be able to access the world of numbers and to learn how they can explain the world around them.

Try reading '*Mathematical mindsets*' by Jo Boaler (2016, Jossey-Bass) as inspiration.

Why aren't primary maths lesson fun any more?

The historical routes to this commonly asked question are explored by Mark Pepper. This piece will be of great interest to primary colleagues who are struggling to navigate the minefield of initiatives that face them as classroom practitioners.

If one considers the current situation within the teaching of mathematics in the primary sector, it may be seen that a tension exists between what teachers would choose to teach and what actually takes place in the classroom. There appears to be a strong consensus amongst mathematics teachers that they would like to teach a balanced curriculum comprised of the learning of new skills, the consolidation of skills already encountered as well as the use of problem solving activities, investigations and activities that are likely to generate enjoyment for the learners. Nevertheless in reality there appears to be a heavy preponderance on the rote learning of number facts and the application of taught algorithms (commonly with the use of a whole series of questions aimed at reinforcing the application of a single algorithm).

In order to try to understand how this contradiction has arisen it is helpful to consider the heavy influence of Government policies within the past 35 years or so. It may be seen that there has been considerable ebb and flo between initiatives that have promoted a heavy concentration on the teaching and learning of a traditional approach and those that have encouraged a more progressive approach. The most influential initiatives in chronological order have been The Cockcroft Report, The Mathematics National Curriculum. The National Numeracy Strategy and various revisions to the National

Curriculum. The introduction and continued use of National Tests and the use of the results to make assessments on both individual teachers and their institutions have had a heavy influence on the content of mathematics lessons.

The Cockcroft Report (1982)

A truly revolutionary change in teaching practices within mathematics emerged with the publication of the highly influential Cockcroft Report. The central recommendations are encapsulated in Paragraph 243:

1. Exposition by the teacher.
2. Discussion between teacher and pupils and between pupils themselves.
3. Appropriate practical work.
4. Problem solving including the application of mathematics to everyday situations.
5. Investigational work.

(Notation added for ease of reference.)

A consistent theme in the Report is that pupils should consider mathematics to be an enjoyable subject

In the years immediately succeeding the Cockcroft Report, schools implemented all of the recommendations. The first of these would already have been established for many years within traditional teaching. 2 and 3 would already have been in force as a consequence of the influential *Plowden Report* (1967) with its strong advocacy of interactional group work and the widespread use of practical work and discussion. The main innovations occurred within 4 and 5. During this period and for several years after it I taught in three successive primary schools in the role of classroom teacher/ mathematics co-ordinator. Additionally I visited other classrooms and had many discussions concerning the teaching of mathematics with a number of teaching colleagues. In my experience most teachers enthusiastically embraced the innovations with a regular weekly investigation, a problem solving approach, a substantial use of practical resources, encouragement of discussion and the regular use of maths games.

A consistent theme in the Report is that pupils should consider mathematics to be an enjoyable subject. Paragraph 347 applies this to the primary sector:

The challenge for the teacher is to present mathematics in a way which continues to be interesting and enjoyable.

One way in which this was achieved was through the regular inclusion of maths games. In my experience this generated great enjoyment for the pupils as well as reinforcing skills learnt through a more formal approach.

The Mathematics National Curriculum (1989)

The effect of the introduction of the mathematics national curriculum (NC) was that for the first time teachers had a statutory obligation to teach specific skills and encourage the learning of specific facts. Most of the content would already have been in the process of being taught in primary schools. The welcome additions to this consisted of a considerable increase in data handling activities and the introduction of probability. A further positive innovation involved the introduction of two Using

and Applying Attainment Targets (ATs) following a recommendation in the Cockcroft Report. AT 1 applied to number, algebra and measures and AT 9 applied to shape and space and handling data. This was a crucial development as it meant that pupils would need to be directed to apply mathematical skills that they had acquired to problems that they encountered. Additionally a provision was made that pupils should independently select the resources needed to respond to questions. A disappointing aspect of the National Curriculum was that it failed to make the regular use of mental maths a statutory requirement. (Though it was included in the non-statutory section).

The booklet that covered mental maths was particularly useful with teaching ideas that could readily be used in a classroom setting

Revisions to The Mathematics National Curriculum (1991, 1995 and 2014)

The effects of the various revisions of the NC was to re-balance the content towards a greater concentration on the rote learning of facts and skills leaving fewer opportunities for engagement in the creative, problem-solving aspects of the curriculum. The 1991 NC reduced the number of Using and Applying ATs from two to a single generic AT that covered every aspect of the mathematics curriculum. This AT was inexplicably abolished altogether in the 1995 NC. The 2014 NC increased the rote learning components with the requirement to memorise Roman Numerals and to know multiplication tables up to x 12.

it is unlikely that mental maths is in prominent use in many primary classrooms

The National Numeracy Strategy (1999)

With the introduction of the National Numeracy Strategy (NNS) the prescriptive aspects of teaching mathematics were intensified. The consequences of this were that in addition to the requirement to teach specific skills the NNS directed that each component should be taught within a tight time scale. Hence a teacher was not empowered to use her/his professional judgement to decide when a class had a sufficient understanding of a concept to deem it appropriate to move on to a new topic. Instead the topic had to be terminated in accordance with a rigid schedule contained in the NNS.

A positive aspect of the NNS involved the introduction of the Daily Mathematics Lesson (DML) which included a directive that the opening 10-15 minutes of each lesson should be devoted to a mental maths starter. This provided a useful opportunity for discussion comprising methods of mental computation as well as enabling the teacher to correct any misconceptions. Furthermore it provided a useful means of informally assessing the progress of members of the class. It also provided opportunities to use activities that helped in the learning of number facts such as multiplication tables.

A most impressive aspect of the NNS arose from the training resources that supported its introduction to teachers through the means of training videos and

booklets. The booklet that covered mental maths was particularly useful with teaching ideas that could readily be used in a classroom setting. This booklet would still be a useful resource for primary teachers today.

The NNS was withdrawn in 2011 and so the delivery of the DML was no longer an official requirement. The regrettable consequence of this is that it is unlikely that mental maths is in prominent use in many primary classrooms.

The Effects of Current Assessment Procedures

The main means of assessing the mathematical attainment of primary pupils involve the use of National Tests (commonly incorrectly referred to as SATs). Pupils are required to undertake these tests in Year 2 and in Year 6. There have been reports that they have generated considerable levels of anxiety for the pupils especially in instances in which the parents and/or the school have applied pressure on the pupils in the quest for acceptable results.

the use of maths games appear to have virtually disappeared

With reference to the content of mathematics lessons the tests have also had the effect of discouraging the widespread inclusion of creative activities. Firstly this is the case because there is such a vast quantity of material in the Programmes of Study, on which the tests are based, that there is extremely limited time available to cover activities that do not consist of the learning of facts or the application of taught algorithms. This is further exacerbated by the fact that the results of the National Tests play a central role in the assessment of teachers. The consequence of producing National Test results that are considered to be unacceptable is that the teacher is liable to be placed in special measures. Furthermore If the results of a school are considered to be unacceptable then there is a real danger that the school will be categorised as a failing school.

The consequences of current mathematical policies

Regrettably many of the activities that generated past enjoyment of mathematics have now to a large degree been discarded. I do not have empirical evidence to support this assertion but there is plenty of anecdotal evidence. In the course of a number of discussions with present primary teachers and also with primary pupils it has become evident that the use of investigations on a regular basis and a general problem solving approach have largely been withdrawn . Furthermore the use of maths games appear to have virtually disappeared. The following two examples are an indication of the present position.

George

I make occasional contact with the son of a family friend who is in Year 2 and I will refer to him as George. When he was in Year 1 he enthusiastically told me that maths was his favourite subject. I saw him recently and he showed me his maths book which consisted of work that he had done up to that point in Year 2. This included lengthy exercises entirely devoted to quite complex decomposition questions. It was clear that George had spent a considerable amount of time in unsuccessfully trying to answer the questions. I asked him if maths was still his favourite subject and he replied:

“I hate maths now.”

Jack

I taught Jack at a school for children with a visual impairment in the mid- nineties. Jack had shown enjoyment for maths lessons in general and for maths games in particular. He is now in his early twenties and is employed at the same school as a classroom assistant. I spoke to him recently and he referred to current maths lessons at the school:

"It's all changed now. There are never maths games like there were when I was at school. There's no fun any more."

Remedies aimed at reversing the effects of current policies

The current unsatisfactory state of affairs could be largely reversed with the implementation of the following:

- Produce lessons in accordance with the recommendations contained in Para 243 of The Cockcroft Report
- Implement a comprehensive training programme for primary teachers delivered exclusively by highly qualified, experienced maths teachers.
- Abolish the use of National Tests. If NTs were to be retained then sever the links between the results of the tests and teacher/school assessments.

Mark Pepper

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The National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6 (1999) DFEE

The National Numeracy Strategy: Teaching mental calculations (1999) QCA

Mathematics in the National Curriculum (2014) HMSO

Encouraging learners to enjoy mathematics – Maths is fun

Mary BJ Clark has many years of experience supporting all types of learners. Here she reflects upon the value, and place of games, within the mathematics classroom.

Many teachers want to counter the all too common tendency for those learning mathematics to lack confidence in their ability to do so. They want learners to see mathematics as fun. An interesting question that this raises is whether having fun should be passive or active in an educational context. For example watching a film is passive whereas playing a game is active. To encourage active fun the use of games in mathematics lessons has a central role. Games have the potential to be engaging and enjoyable which may not be adjectives that describe the common expectation or experience of

those studying mathematics.

So why use games as part of mathematics pedagogy?

1. Games engage learners in mathematics.
2. They encourage children's reasoning and problem solving.
3. They provide contexts for consolidation and practice of skills and application of knowledge.
4. They provide an environment in which children question and get quick feedback on their choices.

In playing games as part of their mathematics learning children experience;

- Active involvement,
- Success and consequent satisfaction,
- Growing self-confidence,
- Enjoyment,
- Excitement,
- Increased interest.

For the teacher this is an excellent context for assessment, so let's have a look at a couple of games. A favourite of mine, one I have used with many classes and with a little tweaking it can be varied to suit all, is called Numero.

Numero - a place value game

7	3	5

9	5	2	4

	8		5		1

Ask the children to draw a set of boxes in a row (you can use any length of row depending on the children you are working with - three, four or five can be a good choice). Have a set of all ten digits available from which to draw and select digits at random, always from all ten digits. The digits can be on pieces of card or drawn on counters or ...
The children have to write the numbers into their boxes as they are called and once a number has been written in that decision must be kept.

As the leader of the game you can decide on an appropriate target - maybe the largest possible number, maybe the smallest, maybe the number nearest to ... or a number between ... and ... . Whatever the goal you set, the children have to make decisions based on place value and some useful discussion about what numbers were possible and their relative sizes can follow each round.

The level of excitement can be increased by a provocative scoring system. An example of such a system would be if the target were the largest number possible then 24 is scored, two 12, if three 8, if four or five 6, and so on with only 1 scored if most people get it right.

In *Equals* 20.1 I revisited the Primary National Strategy resource: *Wave 3 Mathematics resources: Supporting children with gaps in their mathematical understanding*. These resources include a wealth of games both within the teaching materials and in the assessment activities at the end of each error/ misconception section.

Here is an example of a game from the addition and subtraction materials 3Y6+/-

3Y6 +/−

Spotlight 5: a learning check

Has difficulty in partitioning numbers with zero place holders and/or numbers less than one, for example partitioning 0.45 as 0.4 and 0.05

Opportunity for: explaining and discussing

Zap the zero

Time 5–15 minutes

Resources

- One calculator for each pair
- At least two children
- Numbers on slips of paper that have one zero in them (609, 2053, etc.)
- Timer

Check: does the child use key vocabulary?

zero	units/ones
place holder	decimal
column	tenth
hundreds	hundredth
tens	partition

Teaching activity

'This game, **Zap the zero**, will help you with understanding more about place value and numbers with zero in them.'

Pairs can cooperate to play this against another pair, each pair trying to get their score to one whole one in ten minutes.

You need to give each child or pair some numbers with zeros to get them started, doing this secretly so the other children don't hear. For example, one child or pair might have the number one thousand and sixty-three on paper, and they must know the name of the number and how to key it into the calculator.

How to play

1. The first child enters a number that has at least one zero into the calculator. They must read the number correctly and pass the calculator to the other pair or child, for example 1063 read as one thousand and sixty-three.
2. The second pair or child must in one move (either the [+] or [−] key, a number and the [=] key) make the zero become another number.
   For example, 1063 + 900 would make 1963. The zero has disappeared so that second child or pair score 0.1, but they must also read the number they have made, in this case: one thousand, nine hundred and sixty-three.
   If this is done correctly, that pair score 0.1.
   If there is still a zero in the number, they score nothing.
3. Then the second pair or child enters a different number with a zero in it and so on.
4. Aim to get a score of one whole one (ten tenths) in ten minutes.

Variations

- Play with decimal numbers. This could be just tenths or hundredths as well, for example keying in 6.01.
- Play with numbers that have two zeros in them.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- partition numbers with zero place holders, including decimals with tenths and hundredths.

See either of the following links for the full set of resources:

https://www.stem.org.uk/elibrary/resource/29225

http://webarchive.nationalarchives.gov.uk/20110202093118/http:/nationalstrategies.standards.dcsf.gov.uk/node/20314

I have found it interesting to reflect on my assertions about what games have to offer for mathematics learning at the beginning of this article. In terms of pedagogy both games provide the opportunities I mentioned and in my experience have encouraged learners to see maths as fun.

Postscript: I have found it fascinating at the end of a lesson, which included games, puzzles etc. to ask children what mathematics they have been learning during the lesson. In my experience, it's quite likely that they will be a bit confused and assert that they haven't been doing mathematics! So I wonder what their expectations of mathematics lessons are?

Maths talk

Taking time to think about graphs.

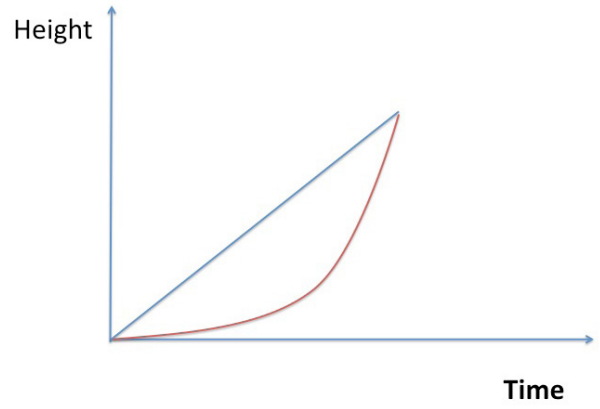
Rather than another interview I thought I would highlight a conversation that took place between three Year 9 students.

I am carrying out some research into the conversations children have in science lessons and part of this involves me using a Gopro camera to record them talking as they work in small groups. The video footage provides a real insight into the focus of their conversations when challenged and when no teacher is present.

Just before Christmas, due to an issue with resources, I ended up teaching a lesson that sought to engage them in thinking about different patterns of mathematical growth.

The conversation in question is from the start of the lesson where the pupils are challenged to think about what the lines on a graph might be showing. It is only a small excerpt but I feel it gives an insight into how adolescents think about graphs.

The following two images were used and introduced with the story in italics as a means of introducing the lesson.



"Imagine a fair visits your town one summer's day. You decide to go and have two rides to choose from, a rocket or a balloon. On this day there is no wind at all and both rides go straight up. Look at the next slide which shows a graph, I am not going to show it for very long, and decide which line represents the balloon and which represents the rocket."

The context of the lesson i.e. real objects moving across the sky is chosen to challenge the children to realise that graphs are not concrete pictures of objects in space. It is designed to get them to talk about the fact that the lines represent the relationship between two or more variables; here time and distance covered i.e. speed.

The graph slide has only been shown for 2 seconds and as the children talk they pause, when their words falter, and move their hands around a great deal.

Please read the conversation with the following questions in mind:

1. What is happening as the children talk through the problem?
2. How does the conversation develop understanding of what the lines are showing?
3. Who helped H to move forward?
4. How were they viewing graphs before this activity?

H – The rocket goes straight up ….. it shoots off ….. the balloon just kind of floats up and goes in any direction.

T – Isn't that line a lot steadier though?

Question from the teacher – What is the graph telling you? What might the lines mean?

H – I still think the balloon is the red one … cause it's showing the red took longer to go up. The rocket would just shoot up but the balloon would float up and it takes longer. A rocket would not go straight up.

L – Yeah but which is faster?

H - Don't know … is it the balloon?
Question from the teacher – What would help you to know which is which?

T – The rocket must take longer to take off, as it needs to build up speed. The rocket goes boom (he now imitates a rocket taking off).

L – Yeah I can see now, the balloon just starts going up and stays pretty much at the same speed. (Moves hands in diagonal line).

H – Ahh …. that makes senses actually … yeah okay …. the balloon stays at the same speed but the rocket does not.

L – The rocket takes longer to take off and when it

gets to speed it just goes straight up.

H – Yeah

H - Because we always just look at it and think a straight line graph means something is straight. But that lines means the balloon is going at the same speed.

If anyone would like to use the activity then please email me (edmiston01@btinternet.com) and I will share the resources and explain the lesson. After this start it goes on to challenge learners about the nature of exponential growth.

I would like readers to share the activities and strategies they use to help their students to think more clearly about graphs. Please email your ideas to me using the address above.

The teaching and learning of mathematics by pupils with a VI Part 2

Mark Pepper carries on where he left off in the second part of his analysis of the way forward for teachers of students with VI.

Introduction

The extent of the difficulties for learners with a VI (particularly those who are completely unsighted) in the learning of mathematics are considerable. Activities associated with Shape, Space and Measure pose the greatest difficulties. The purpose of this article is to try to convey the extent of these problems and then to suggest resources and teaching strategies aimed at trying to overcome them as far as possible.

The focus will not be on a particular Key Stage (KS) as different aspects will be relevant to different age groups. In some cases an indication of the relevance of a particular KS will be given. This will, however, be a fairly loose indicator as the material could also be relevant to other age groups. For example students who have learning difficulties will commonly be operating with material from an earlier KS than that normally appropriate for their age group.

The format will be in accordance with the attainment targets (ATs) contained in the Mathematics National Curriculum.

AT 1 Using and Applying Mathematics (obsolete)

One of the strongest components of the original mathematics National Curriculum was the inclusion of Using and Applying mathematics as it enabled teachers to provide activities that reinforced skills learnt within the other ATs. Furthermore it provided opportunities for learners to independently select the resources to tackle a problem. It also legitimised

activities likely to make mathematics an attractive subject with the liberal use of investigations, problem solving and maths games.

Whilst the delivery of Using and Applying mathematics is no longer a statutory requirement it does nevertheless seem eminently sensible to continue with its use as far as possible. In a VI context the continued use of AT 1 is especially appropriate as the use of discussion of mathematical problems is particularly helpful for learners with a VI.

AT 2 Number and Algebra

Mental maths

The use of mental maths in maths lessons involving learners with a VI is especially helpful as it avoids the use of enlarged work sheets, tactile diagrams or brailled materials which can be a cumbersome process. Apart from this, mental maths is a useful means of giving learners the opportunity to discuss different methods of computation and it also provides opportunities to reinforce the learning of number facts such as multiplication tables. When the Daily Mathematics Lesson (DML) was in force teachers were required to open every maths lesson with a mental maths starter. The consequences of the demise of the National Numeracy Strategy and the resulting abandonment of the DML has resulted in there no longer being a mandatory expectation for the regular provision of mental maths. Nevertheless it seems sensible to make liberal use of it anyway, particularly in a VI context.

mental maths is a useful means of giving learners the opportunity to discuss different methods of computation

Tactile Resources

Key Stages 1 and 2

It is essential to have a wide range of tactile materials available for pupils with a VI. This is particularly useful within activities such as counting and within the four number operations of addition, subtraction, multiplication and division. The following are useful resources:

Multilink cubes

Multilink cubes are ideal for use by pupils with a VI as the raised circle on the top of the cubes enables learners to easily distinguish each individual cube and thus facilitate the counting process. This is in contrast to the use of Unifix cubes in which it can be difficult to feel the "join" of adjacent cubes.

Sum things

Sum things consist of a string of ten coloured beads. They are a helpful resource for activities such as counting and for procedures that reinforce the learning of number bonds to ten. The particular strength of this resource is that when the string is held upright the beads do not all descend to the bottom of the string. Each bead can be moved individually and it will then remain in a fixed position. This means that in an operation such as 6 add 2, the learner can carefully count 6 beads and move them to the bottom of the string. A further 2 beads can then be pulled down to join the 6 beads. The learner can then count all of the beads at the base of the string to give a total of 8.This resource

is particularly appropriate for a student with VI who has multiple disabilities. These students commonly have poor fine motor skills and the manipulation of Sum things is much easier than the use of multilink.

Pegs/pegboards

An array of pegs on a peg board can be easily felt by a pupil with a VI. The fact that the pegs remain in a static position is also helpful.

Dice

Dice are available both in tactile form and in enlarged format known as jumbo dice. These can be useful in activities such as counting, addition and multiplication.

Number lines

Number lines are a useful resource for activities such as addition, subtraction, multiplication and division. In addition to a large communal number line on display in the classroom, some teachers also provide each pupil with an individual small desk top number line. In these cases a child with a VI should be supplied with an adapted version in either large print or in brailled format. In instances where desk top number lines are not provided it is essential to provide a child with VI with an appropriately adapted number line as they would not be able to access the classroom number line. The use of number lines is not straightforward for unsighted pupils. For sighted pupils the number line provides a constant reinforcement both of the number symbols and of the ordering of numbers. The immediacy of this reinforcement does not exist for unsighted pupils who would be obliged to laboriously feel each individual number symbol and thus lose any continuity. Furthermore sighted pupils can rapidly count on or back on a number line whereas unsighted pupils need to identify a starting number with the use of touch and then count on or back by separately feeling each number.

The use of number lines is not straightforward for unsighted pupils.

1-100 Number Squares

1-100 number squares are available in large print format and in braille. This is a useful resource to reinforce the ordering of numbers and for activities involving addition and subtraction. Unsighted pupils are subject to similar disadvantages to those associated with the use of number lines.

Jumbo calculators

Calculators with enlarged keys and display are available and these are of great benefit to learners with a VI. One issue that can cause confusion involves the decimal point. There are occasions when a learner can see the number symbols on the screen but does not see the decimal point. Hence a number such as 51.2 can be interpreted as 512. This difficulty is compounded by the fact that the learner will be unaware of the existence of the decimal point as opposed to being uncertain of the identity of a number symbol in which case clarification can be

One issue that can cause confusion involves the decimal point.

sought from a sighted person.

When the learner is supported by a 1 to 1 teaching assistant (TA) it is essential that the TA records the decimal point with a fluorescent pen or in enlarged form with a thick black pen when recording numbers for the learner.

Talking calculators

The talking calculator is a useful resource for braille users. These pupils are, however, at a disadvantage due to the wording of the automated voice in announcing 3 digit numbers.

E.G. 109 will be presented as one zero nine as opposed to one hundred and nine. Whilst the latter mode makes clear the place value of each integer, the former does not. Whilst it is true that adjustments to the settings can be made on the calculator such that that the automated voice presents a 3 digit number correctly, this will only happen for a single reading and then the calculator will revert back to its original mode. In practise it is not worthwhile to have to constantly make adjustments to the settings and so it is usual for learners to dispense with use of the adjustment.

K.S. 3 and 4

It is an unfortunate fact that some standard talking calculators have deficiencies. The most serious of these involves the lack of a square root button! It is true that scientific talking calculators have similar functions to those of standard scientific calculators which, of course, includes a square root button, but they are expensive. A standard Reizen talking calculator stocked by the RNIB, costs £9.95 whilst the cost of a SciPlus 2300scientific talking calculator is £290. The consequence of this substantial difference in price has led to some schools being unwilling to invest in a scientific calculator if it is for a student who will sit the GCSE Mathematics Foundation Paper. Such a student would then be without the means of electronically finding the square root of a number.

A further facility missing on many standard talking calculators is a constant button. Hence learners using these calculators do not have a short cut for repetitive addition or a means of rapid reinforcement of multiplication tables.

Fractions

Sighted students are accustomed to a standard format in the presentation of fractions. This form of display is helpful within the multiplication and division of fractions as cancelling of numbers can easily be undertaken by a competent learner.

It is an unfortunate fact that some standard talking calculators have deficiencies.

e.g. One third multiplied by three eighths would be recorded in the following notation:

$$\square\ \frac{1}{3}\ \times\ \square\ \frac{3}{8}$$

The nominators and denominators are immediately apparent and processes such as cancelling by 3 can be readily undertaken.

The braille user does not have this facility as the layout of fractions in braille is completely different. In braille the above example in its entirety would

be contained in a single horizontal line. Hence references to 3 over 8 would be meaningless and processes such as cancelling could only be achieved by holding the numbers in the head.

AT2 Shape, Space and Measure

The disadvantages for learners with a VI (especially those who are totally unsighted) within this A.T. are substantial. Some of the difficulties are fairly obvious whilst others are not immediately apparent.

The interpretation of a diagram

Whilst a sighted student can at a glance see a diagram in its entirety, the student with a VI is obliged to feel parts of the diagram in piecemeal fashion and then try to imagine the image in its entirety. It is remarkable that many students become adept at achieving this.

One of the main traditional resources within mainstream classes has consisted of a set of Attribute Blocks.

Construction of a shape

The act of constructing a shape is fraught with difficulties for students with a VI. The main mathematical tools for this, such as rulers, protractors and compasses are available in adapted tactile form and the learner is supplied with a cork board and pins. A material, colloquially known as German film, is appended to the board with the pins. The learner can then make use of the adapted resources in an attempt to construct the required shape. Quite clearly this is a far more complex task than that of a sighted student who could undertake the task with the use of conventional resources.

Practice in constructing a 3D shape can be achieved with the use of Geoshapes. This consists of a set of 2D shapes which interlock.

Shape

2D Shape

It is essential to have a bank of tactile resources available for learners with a VI. One of the main traditional resources within mainstream classes has consisted of a set of Attribute Blocks. A set of enlarged pieces known as Giant Attribute Blocks are also commercially available. This is a useful resource for pupils with a VI and consists of squares, rectangles, circles, triangles and hexagons. An Attribute Block can be passed around a group and each pupil can feel it and describe its main properties. A weakness associated with Attribute Blocks is that all of the triangles are equilateral and this could lead to a misconception that all triangles are equilateral. Alternative resources involving 2D shapes are commercially available. Many of these sets include a fairly wide range of 2D shapes including ovals, semi-circles and a variety of triangles including right angled and isosceles triangles.

One difficulty in the use of tactile resources in the teaching of 2D shape is that there is a continuous contradiction in that 3D shapes would be being used to represent 2D shapes. This can lead to a considerable amount of confusion amongst the pupils.

3D Shape

In many primary classes a resource used in teaching

3D shape consists of a commercially produced set of 3D shapes. Instead of investing in this resource an adequate set of 3D shapes can be obtained from "junk" material as follows:

Cube	Sugar lump or dice
Cuboid	Empty carton of tea or cereal
Prism	Toblerone packet
Cylinder	Smartie tube
Sphere	Ball
Pyramid	Pyramid tea bag

For pupils with a VI there is a further advantage in the use of these materials as in addition to experiencing the feel of 3D shapes they would be interacting with objects from their everyday environment.

Measurement

Length

The measurement of a line can be achieved by learners with a VI with the use of a 30 cm tactile ruler. This resource has a yellow background with black markings. Each centimetre is represented by a ridge on the back of the ruler. Measurement can then be undertaken by counting the appropriate number of ridges.

The main resources that are likely to be required consist of tactile squared paper and a set of Wikki Stix.

For tasks such as measuring the length of the classroom a talking tape measure could be used.

To measure the length of longer distances such as the length of the school field a trundle wheel could be used. At each revolution of the wheel there would be a distinct click which would indicate that one metre had been covered. The student could continuously count the number of clicks to determine the distance that had been covered.

Angles

The size of an angle can be measured with the use of a tactile protractor. This device has a yellow background with black markings and contains a series of ridges, each of which represents ten degrees. The size of an angle can be determined by counting the appropriate number of ridges.

Data Handling

Many of the activities associated with data handling do not present particular difficulties for learners with a VI. The main resources that are likely to be required consist of tactile squared paper and a set of Wikki Stix. The latter consists of strips of material that are sticky and can be appended to paper. They then remain in place due to their adhesive qualities.

Conducting a survey

The stages within the process of conducting a survey could be achieved by a student with a VI as follows:

Collection of Data

Within the process of the collection of data, the student with a VI could be partnered by a reliable sighted student. The former could verbally ask the respondents questions whilst the sighted student could initially record the responses in writing. At a later time the student with a VI could then record

the data on a computer or with the use of braille.

Tally Charts

When the construction of a tally chart is required an unsighted student could use a brailler for this purpose by representing each event with a single dot. Each set of 5 dots could then be recorded by brailling a double dot.

Construction of Bar Charts

Bar charts can be constructed with the use of tactile squared paper and Wikki Stix. The outlines of the various rectangles at the appropriate heights could then be represented by covering the outlines with pieces of Wikki Stix.

Interpretation of graphs

The graphs would need to be presented in tactile form. The student would then need to feel the outlines in order to interpret the graph.

Presentation of Data

It can be helpful to make a display of the graphs constructed by the class and then invite other classes to visit in order to interpret them. Any tactile graphs could, of course, be included in this process.

Activities such as these help to create an atmosphere of inclusiveness and mainstream students are likely to be interested in the creation of graphs by students with a VI.

Maths Games

A most undesirable consequence of the abolition of AT 1 and the unrelenting pressure placed on schools and individual teachers to produce acceptable results from their pupils within National Tests examinations has led to the virtual elimination of maths games in the classroom. They flourished in the immediate post Cockcroft period and for a number of years beyond it and they provided an effective means of making mathematics an attractive subject for young learners. Additionally they helped to reinforce skills that had been learnt in other maths lessons. Within a VI context games such as *Beetle* and *Shut The Box* have been adapted into tactile versions that could be played by learners with a VI. *Beetle* is a game appropriate at KS 1 that involves counting skills. *Shut the Box* is an ingenious game that involves practice within the four number operations. Additionally an impressive board game named *Jody's Pegs* was produced by a VI advisory teacher called Jenefer Dunnett for the specific purpose of being available in maths games sessions for one of her pupils who had a VI and was called Jody. *Jody's Pegs* consist of a series of stimulating strategy games with the use of specially designed large pegs and a peg board. In my experience all of these games were a source of great enjoyment that also reinforced various maths skills. Additionally they greatly facilitated a

Additionally they helped to reinforce skills that had been learnt in other maths lessons.

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natural process of inclusion as sighted pupils were invariably eager to be involved in playing these games.

It is extremely depressing to report that at a recent visit to the RNIB shop I discovered that all of these games had been discontinued.

Whilst it has to be reluctantly accepted that in the current educational climate it is not feasible to have regular maths games sessions within maths lessons it is to be hoped that they could still feature in schools. Opportunities for this could arise at times such as the last day of term or during wet play. Additionally perhaps an after school club could consist of traditional board games such as Chess, Draughts, Scrabble and Snakes and Ladders as well as the VI games referred to here.

Mark Pepper

Mark Pepper was Head of Mathematics at Linden Lodge School for blind and visually impaired children from 1996-2006

References

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Harry Hewitt Memorial Award

This prize is awarded to any pupil who has overcome barriers with mathematics and is now making real progress.

Do you have a pupil, like this, who has struggled but is growing in confidence as they engage with mathematics? Why don't you celebrate their success in *Equals*?

We are offering a prize of a £25 book token to the best entry we receive and the opportunity to have the work published in *Equals*.

Choose a piece of work that both you and your pupil consider successful and send it to *Equals*. Please include:

- the original piece of work, photograph or photocopy,
- an explanation from the teacher of the piece of work and its context and a description of the barriers which the pupil has overcome or is in the process of overcoming.
- the pupil's age, school and the context of the class in which they learn and, if possible, some comments from the pupil about what they are pleased with about this piece of work and/or the learning it shows.

Entries should be sent to Alan Edmiston (edmiston01@btinternet.com) by the end of April.