Equals

Realising potential in mathematics for all

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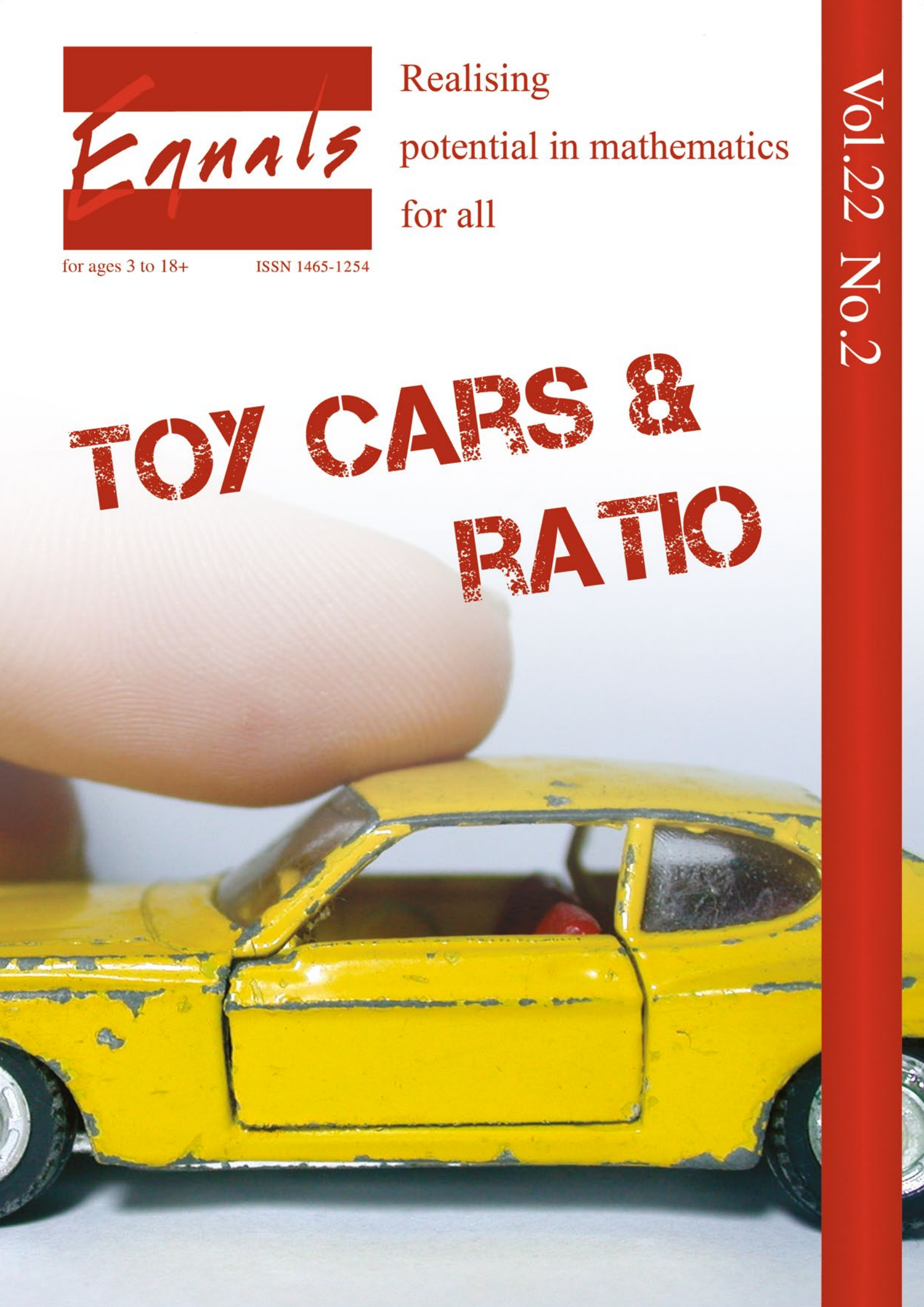
TOY CARS & RATIO



Equals

Realising potential in mathematics for all

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Designed by Nicole Lane

Editor's Page

This edition of *Equals* contains two activities, suggested by me, for you to try in your own classroom. They are both lessons I love and have tried in a wide range of schools. **Fun with Ratio** is something I have been developing for a number of years now and if you wish to try it please email us for the resources and other supporting materials. This activity is aimed at KS 3 pupils. **Where are the Frogs?** is something I taught in Charlton Park Academy a few weeks back. It can be used with very special needs pupils as a game which serves to help them develop their positional language and ultimately to think about the origins of the co-ordinate system.

Equals has always been at its strongest when practitioners share what they are doing to help those who struggle with mathematics. **Reflections on a weekend in Surrey** contains the thoughts of Pete Jarrett who was able to represent *Equals* at the recent MA Conference over the Easter holidays.

Two articles are very important for me as they carry on the theme of making mathematics fun from the previous edition. Both Pete Jarrett, ***Equals Online***, and Mark Pepper, **Fun Activities**, look back through previous editions of Equals to highlight how fun within the mathematics classroom has always been a strong theme running through this publication. They also highlight what a useful resource of activities and ideas our on-line archive is.

Math's Talk comes from Joanne Thornton of South Tyneside who shares her thoughts on how to support low ability pupils within mathematics, among other things.

Can I remind you to please respond to Mary Clarke who uses this edition to re-launch the Harry Hewitt Memorial Award. I had the great honor of visiting a school to present this award several years ago. The aim of the prize is to celebrate the success of those who are rarely acknowledged. Please read her piece and reflect upon which of your pupils have made a breakthrough which has given you reason to celebrate and do nominate them.

As I write my mind is full of the preparations I still need to do for two SEND Conferences I am soon to be involved in. Both have been organised by Barbara Rodgers of the Solent Maths Hub and take place on the 28th and 30th of June at Swiss Cottage School, London. Both Barbara and her colleague Julia Brown have been kind enough to make hard copies of the last edition of *Equals* available for all participants on both days. I am looking forward to sharing, and promoting, the work of *Equals* with the delegates and I am sure that future editions will be populated with ideas from classroom around the country.

Feedback from the MA Easter conference has given us the focus for our Autumn edition. All of the articles will focus on anxiety and how to support learners whose dislike of math's results in some very negative responses and behaviours. If you

have any questions you would like us to answer or any stories to share then please email us and they will be included.

In the past my own involvement in *Equals* has been to share activities I have tried in the classroom with lower ability or SEND pupils. In this edition I have finally put pen to paper and write about two activities I tried last term. Sharing these is important as a shared classroom focus helps us to clearly articulate the issues associated with teaching lower ability pupils. One of the lessons has already been tried by a colleague from North Tyneside. Sam tried the Frogs lesson with her Year 7 group and was able to reflect that:

'some of the kids are more mathematically able than I thought which begs the question how do I help those kids without them struggling with their other difficulties such as dyselxia especially with worded questions. The difficulty is balancing the understanding and the "attainment" of the students. I want the kids to have enjoyable lessons which are relevant and aids their understanding but ultimately we need to preparing these kids for assessments and GSCEs. Getting the balance right is difficult!'

We need a forum for sharing ideas about how we help all pupils and hope you will agree that *Equals* is ideally placed to support such conversations. I will end with repeating my appeal from the last edition – please get in touch to share the things you have been doing in your classroom to support the learning of your pupils. All hints, tips and ideas will be gratefully accepted and shared.

Fun with Ratio!

In this article Alan Edmiston shares an activity on ratio that he has been developing for the past two years. Readers are encouraged to try the activity but more importantly to share the practical ideas and activities they have for supporting their pupils thinking around ratio.

In my role supporting schools around the UK I regularly have the opportunity to try out activities with a range of different pupils across Key Stages 2 and 3. This gives you some very interesting insights into how pupils think about the more difficult concepts within mathematics, especially if you try the same activity with Years 5 and 8!

For the past two years I have been involved in the development of a series of lessons, three of which concern ratio. Here I will share the first activity as tried with two 'bottom' set classes in Year 7, one from a school in Wolverhampton and the other from North Tyneside. Where possible I will include photos to show both the equipment and the ideas that

emerged during the discussions. Pupil comments were also collected from the latter school and will be shared at the end.

I will split the article into two parts:

A. an overview of the lesson split into episodes and,
B. a recount of the thinking that took place during the lesson from both schools.

I use the term episode as to me it sums up the fact that the pupils are exposed to an idea that develops over time. A lesson can be made up of a single episode or up to three or four depending upon the pupils and the length of time available. The activity in question has up to four episodes and I find that most groups manage about three in an hour lesson.

Part 1 – lesson overview.

Episode 1 – ratio as a function of teeth in gears

In this episode the pupils experiment with different pairs of gears to explore the ratio that comes from the relationship between the number of teeth that make up the cogs. This part of the lesson starts with an image of two people riding up a hill. Is it the people or the bikes that make the difference? 2/3rds of Year 7's tend to say its because the male

is fitter while the rest point to the fact that his bike might have gears or that he is in an easy gear. They then experiment by turning two or three different pairs of gears. The idea here is to think about which gear set makes it easier to ride up hill.

Episode 2 – toy cars and ratio

We now move onto the way ratio is used to describe the scale in model cars. As a child I was always fascinated by toy cars and 40 odd years ago when I played with them they all had their scale written underneath. Recently when playing with my nephew I found that many toy cars now do not include the

scale. This episode is built around the fact that my nephews 'Car's' cars do not have a scale yet the ones from my loft do. The pupils struggle to spot the difference then are really challenged to explain what the scale means. This is where mathematics is more than numbers, in answers, and actually tells us something about the relation between two objects here a car and a much smaller model. There are a series of questions growing in difficulty that can be asked during this episode:

- What does 1/56 or 1: 56 mean?
- Some cars have 1:18 or 1:90 written underneath. What is gong on with the numbers?
- What does 1: 1 mean?

Episode 3 – proportion in human beings

For most classes this final part of the lesson challenges them to work out the ratio as a link between arm length and height. Now we are exploring pairs of ratios i.e. proportion and this really pushes their thinking. The scenario used involves someone who needs to be measured but is unable to straighten up. The person measuring says its okay as all they need to measure is the length of their arm from the wrist to elbow and from that the height can be calculated. The challenge for the pupils is to work out how they can do it by measuring themselves and their peers.

The pupils struggle to spot the difference then are really challenged to explain what the scale means

I really enjoy this part as many do realise that they are finding out 'how many arms go into the height' but they think that because we are all different then the relationship is too. They are often taken aback when they realise that everyone is coming up with an answer close to 6. They really struggle to articulate their answers to the following questions:

- What does the 6 mean?
- Why are we all 6?

Possible episode 4 – are toy dolls in proportion?

This lesson can be further developed to include an episode comparing the leg height proportions in pupils and dolls. With older groups this has been used to explore issues of body image and photo-shopping, all of which they find fascinating.

Lesson end – Can you see the ratio?

Usually by this time the pupils are starting to use the word 'ratio' and to articulate what it means now they have had a chance of exploring it in three different contexts. The power point I use for this lesson ends with the title, 'Can you see the ratio?' I use this to send them away with something further to think about. The slides that follow pictorially show the Golden Ratio (Phi or 1.6) in a range of contexts: snail shells,

human faces, fingers, the female reproductive system, Greek buildings and end with the title, 'Did you see it?' They now leave the room animatedly discussing what the pictures have in common.

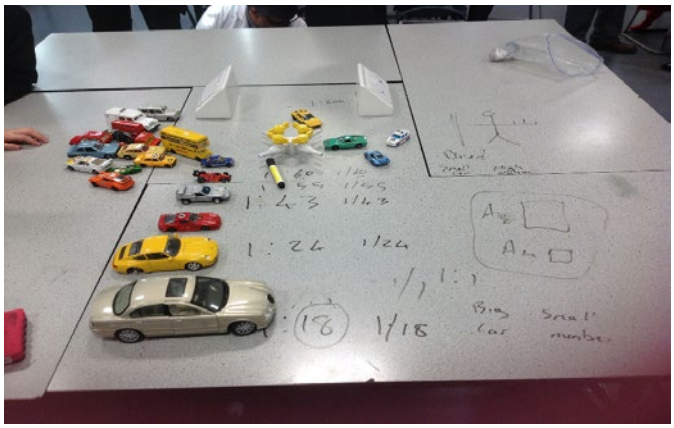
Part 2 - Insights into pupil thinking.

In this section I will share what the pupils spent their time talking about in two of the episodes. This will illustrate the fact that to me the aim here is to listen to how the pupils talk and to move them on in response to the mathematical ideas and thoughts that emerge. Emerge they do but in what form is very hard to predict but what ever is said it is a continual source of wonder and joy. Teaching these two groups was a great privilege and the discussions of the pupils simply highlighted the potential that exists in all learners. 'Ability' was not an issue, yes the activity is challenging but the pupils were responding to a concrete stimulus that served to engage them in deeper thought. In addition to this the context of the lesson i.e. 'What's going on?' served to bringing the real world into the class, which then enabled me to crystallise their math's ideas ready for later use and formalization.

In Wolverhampton it was the cars that provided the spark to ignite the thinking of the class

In North Tyneside it was the size of gears, which resulted in the first and longest animated discussion. They learned quickly and were able to use the notation 1 : 2 to show that if the smaller wheel went round once then the smaller one went round twice. The discussion then turned to which set of gears would be easier to pedal to which one boy wrote on the board 1 : 4. I then drew two gears; the first big and a second very small and when I turned round I could see a pupil at the back of the class thinking deeply about this. I invited him to speak and in a confident voice he said that; "The four is twice as big as the two so the second wheel should be twice as big as the first." I asked him to repeat himself and then invited the rest of the group to respond to his idea. Some agreed with him but many became very animated and began to turn their cogs in response to show the smaller cog going round more or faster than the first bigger one. What was clear now was that many pupils agreed with his idea of 4 being twice as big as 2 but that were now trying to visualise a wheel turning round more times and so having to be smaller to do so. As with pupils of this ability they stumbled over their words and faltered as they spoke to try and explain why he was incorrect in his thinking. But collectively a response began to emerge and it almost became a competition to see who could explain it in the clearest way.

In Wolverhampton it was the cars that provided the spark to ignite the thinking of the class and the photograph below illustrates the focus of our deliberations.



Sometimes in classrooms a little bit of magic happens and that 'moment' lives long in your teaching memory. Such was the case this day, something emerged from the collaboration that could not be predicted and it deserves sharing here. The pupils had really engaged with the cars and realized, after much deliberation, that the numbers on my cars meant something. Given the size of the class we were all able to sit around the bench and organise the cars. Sahir spotted that as the cars got bigger then the numbers changed and that gave the class the focus to lay them out as you can see above. I then decide to write down the numbers and by now they were all able to spot that 'the bigger the car, the smaller the number.' Then something special occurred as three boys began to share their thoughts – each thought was special and unique and was labeled as such:

Then something special occurred as three boys began to share their thoughts – each thought was special and unique

Boy 1 said "That's like paper, I have always wondered why A3 paper is bigger than A4 even though the number is smaller." He was then given the name 'Paper boy'.

Boy 2 said, "All the cars are models of real cars." He was given the name 'Model boy'.

The class were then, with some effort, able to make the link from the first episode that the numbers described how the model car relates to the real car. By then I was writing the numbers down as on the photograph and the third boy spoke up.

Boy 3 eventually said, "If you keep on going you will get to the real car ……. that's 1 : 1". So he was named 'Real boy".

I was then able to ask the class to think about the three boys ideas using their new names. As you can imagine the following discussion and reflection was very rich and powerful. We did not get to the 'Can you see the ratio?' part of the lesson but that fell by the wayside because of how the whole group had engaged with the concept of ratio in a very real, and engaging way. Although I did not ask the class to write down what they thought, their smiles and laughter were enough. The pupils from North Tyneside, part of a project run by the Great North Math's Hub, did and I will end with their feedback upon this lesson:

- *I liked playing and investigating the cars and bikes. It was really fun and enjoyable.*
- *Talking was important as people can hear your ideas.*
- *I liked how you could physically touch the objects and look at it and see the sizes.*
- *We used the cars and gears to help us work out questions.*
- *It helped because you can ask others to see if you are right.*
- *People helped by explaining better so I learned more.*

If any reader would like to try this lesson then please email Alan: edmiston01@btinternet.com

When learners can't or won't do maths – making maths fun and accessible to everyone.

Pete Jarrett **has been reflecting upon the barriers that limit learning and in the process he has compiled this very insightful summary. The article below contains many words of teaching wisdom, ones that all teachers would do well to heed.**

There are many things that can impact on the learning of mathematics. Some relate to the learning environment, and some to the ability to use number. It is not always straightforward to separate out the individual aspects of someone's difficulty with mathematics. This can present a significant challenge to classroom teachers. Over the following pages I will briefly examine some of these complexities and introduce a model for teaching and learning mathematics that offers ways to alleviate or understand some of the barriers. The rich history of *Equals Online* and its predecessors *Equals* and *Struggle* offer many examples of good practice that draw on this model and provide engaging and interesting ways to make mathematics more accessible.

I have ordered my 'barriers to mathematics' purposely. When we are trying to identify what a learner may struggle with, we need to exclude or address barriers as we go. If we can do this in a systematic way, and record our thinking and findings as we go, we are beginning the Assess, Plan, Do, Review cycle that is identified in the current SEND Code of Practice. Should we, further down the road, feel that we have a learner who may have a SEND need, the evidence we have gathered will shorten the time to draw in the correct resources for that learner. The order intends to reflect those barriers which can be fixed more 'easily' at the beginning running down to barriers that exist because of a specific SEND need. The list excludes barriers that have an immediate impact on overall learning, for example visual and auditory disabilities, assuming that those barriers are being adequately managed already.

Mindset and resilience.

Issues around learner mindset and mathematical and problem-solving resilience are high on the agenda now. It is recognised by many, including a number of speakers at Conference, that these are two areas that can impact on success. Whilst not wholly a SEND barrier, learners with SEND difficulties are likely to believe that their mathematical ability is fixed, that they can't get better at the subject and there is little value in trying to do so.

Confidence and anxiety

Whether it is mathematics, mathematics learning or assessment that provokes a response, for many learners there is a definite and disabling emotional affect around mathematics. Emerging evidence shows that learners can be put off mathematics

from as young as four or five years old. By the time they reach 16 and beyond low confidence, anxiety and a fixed mindset lead to substantial barriers to achieving in mathematics. In many cases, learners may present with a different set of behaviours to our normal perception of anxiety - disengagement, absenteeism and poor behaviour can often be linked back to some affective domain issue.

Literacy

Sometimes abstract ideas are quite difficult to put into words, and sometimes it can be very difficult to extract those abstract ideas from words. From functional skills questions, where number and unusual scenarios, often involving village halls, are cobbled together in some mutation of quantity and language to form numglish, to more complex evolutions of etymology, language often makes mathematics harder to understand.

Reasoning

Included, alongside reasoning, are also problem-solving skills and decision-making abilities, all intrinsic to developing mathematical understanding. Weaknesses in reasoning abilities may be due to mindset and confidence issues or to cognitive processing difficulties.

Memory and Speed of processing

Working memory and speed of processing are two of the most important cognitive abilities used in mathematics, particularly in mental arithmetic. Difficulties with these abilities are found across a wide range of Specific Learning Difficulties and other conditions. Difficulties with remembering and recalling facts, answering quickly, getting lost in multi-stage questions and timely completion in exams can often be attributed to difficulties in these areas. This can be compounded by the apparent need to answer questions quickly or remember times tables and number bonds in class, especially when teachers draw attention to these weaknesses, leading to anxiety and lack of confidence.

Arithmetic – learning delays and cognitive difficulty

Weaknesses in understanding how numbers work, or having little sense of number, can present problems in mathematics learning. Learners may struggle with concepts of magnitude and place value, give unrealistic answers or rely on process and long-winded methods. It is important to differentiate between learners who have some form of learning delay and therefore make some progress under intervention and those with cognitive delay that respond less well to well-meant and targeted interventions. Dyscalculia is a specific difficulty around sense of number, and some chromosomal conditions and syndromes such as Foetal Alcohol Spectrum Disorder (FASD) can affect the areas of the brain that deal with our sense of number.

Within this complex set of barriers there is a specific condition that relates to our ability to work with number, dyscalculia. There is no single widely accepted definition for dyscalculia or Specific Difficulties in Maths Learning at the moment. There are two broad models of the condition; the single deficit model, proposed by Professor Brian Butterworth[1] amongst others. This presents dyscalculia as a core deficit, identifiable in the intra-parietal sulcus (IPS) of the parietal lobe (Fig.1), an area of the brain that deals with comparison of number, quantity and magnitude. This sense of number of numerosity is present in most animals and is considered to be innate.

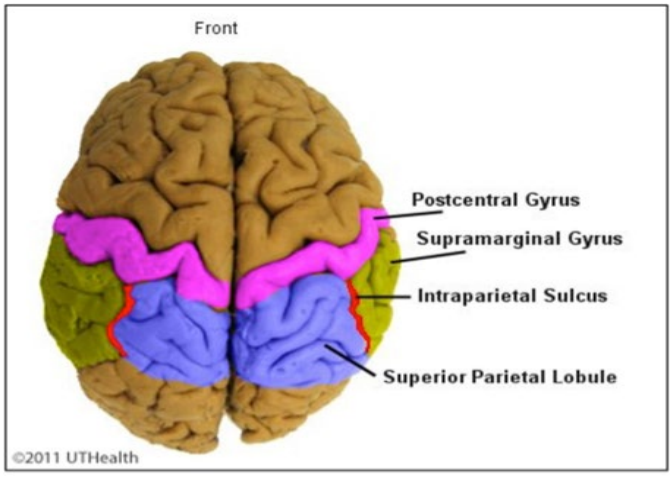
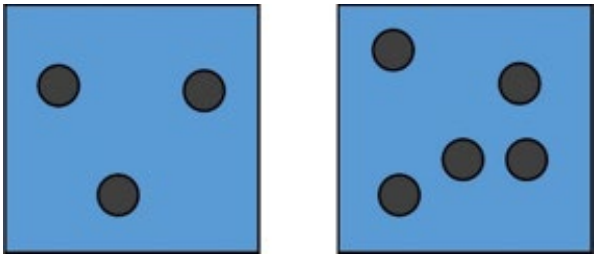


Figure 1. A human brain, identifying the intraparietal Sulcus. Accessed online: http://nba.uth.tmc.edu/neuroanatomy/L1/Lab01p14_index.html

Subitising and estimating

How many dots are there?

We can normally recognise dot arrays of up to four objects without counting - this is known as subitising.



Which group is the largest?

As the number of dots increase, and the number of dots in neighbouring arrays gets closer, it becomes increasingly difficult to recognise which group has more or less.

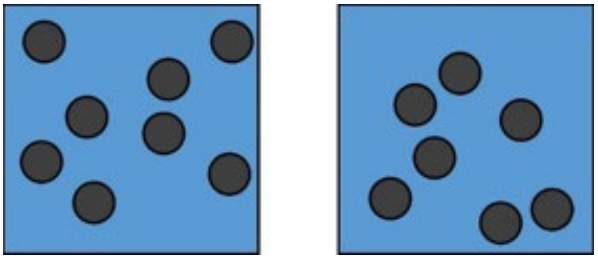


Figure 2. Subitising and estimation. Core skills at the heart of numerosity.

The IPS is involved in skills such as subitising, the ability to recognise small groups of numbers without counting and with dot enumeration, recognising which dot array of two is the largest.

The second model of dyscalculia is that mathematics learning difficulties/dyscalculia are heterogeneous in nature[2] and can be indicated by:

- Logical thinking abilities and counting knowledge
- Language abilities
- Number representation
- Working memory

When learning mathematics, we have to make connections between our non-symbolic representations of numbers, our ability to see the difference between two sets of objects, and our symbolic representations of different number sizes – 4 or 12 or 1 000 000. We first make sense of real things by such skills as counting and our sense of number.

We then use our understanding and language, both verbal and the symbolic language of mathematics, to begin creating abstract ways of describing what is happening when we do stuff with numbers. The symbols we use allow us to explain very complex ideas in a way that is more accessible than just words – but mathematics is a language in its own right.

Mahesh Sharma[3] suggests that there are three components of mathematics – the linguistic, conceptual and procedural. He goes on to suggest a model for levels of mathematical knowledge:

1. Intuitive – that which we already know. The knowledge and understanding that we bring to the table that helps us to make sense of new ideas and information.

2. Concrete – the real-world representations we make of mathematics.

3. Pictorial – the graphical representations we make of the real world of numbers, from simple tallies to more complex graphical models.

4. Abstract – using the symbols of mathematics and language to represent our real world understanding in a more useable or efficient way.

5. Application – marrying our procedural knowledge and conceptual understanding to use our new skills in a variety of contexts.

6. Communication – being able to communicate our new knowledge and skills to others.

The keys to success are clearly understanding all areas that are needed to develop an understanding of a new area. Where cognitive difficulties or gaps in knowledge prevent an intuitive underpinning knowledge then learning will be impeded. Does getting 60% in a test really demonstrate the depth of underpinning knowledge required to move on?

Secondly, demonstrating mastery, by being able to apply knowledge and skills and the communication of mastery and skills becomes an intuitive use of skills in learning the next aspect of mathematics.

At its simplest form, a model that fits with the way we learn mathematics would contain the following elements:

Concrete – manipulatives, real things, stuff that is tangible and that we can get our heads around.

Semi-concrete – drawings, pictures, representations of real things, drawing on our understanding of the tangible to begin to communicate our understanding.

Abstract – working within a more symbol driven approach that allows for efficient explanation of what is happening. As assessments are generally presented in the abstract, it is important that understanding can also be demonstrated in abstract ways.

This model is known as the CSA model. When it is tied in with Sharma's six levels of mathematical knowledge it provides an accessible approach to developing understanding in learners. It is an approach that has been used successfully in the teaching of struggling learners for many years, and it is also the approach that is often used by practitioners when demonstrating engaging ways of teaching mathematics.

Throughout the history of *Equals* and *Struggle* there have been many articles that demonstrate the use of real tools to stretch thinking, develop concepts and provide enjoyment through learning. Many of these articles provide open ended tasks that allow learners to make their own sense of what is happening rather than being trained in the use of an algorithm.

In *Equals 14.1* from 2008, Mundher Adhami presents us with this idea:

What Tree do you Mean?

Story: A gardener and her apprentice have to agree between them on a way of knowing which trees they are talking or writing about in the garden shown here. Ask for suggestions from pairs of pupils talking amongst themselves for a couple of minutes.

The teacher conducts a discussion on different issues in order to generalise:

- What order is possible to use to describe positions?: Top to bottom; left to right; clockwise; left and right; outer and inner.Some pupils may suggest a grid as a space organisation. It is important not to take this as 'the answer', but rather to talk about the pros and cons, or ease and difficulty of different ideas.
- How to classify/sort? Large and small trees; fir and bush, and how the type of tree can help in coding the trees.
- The need for two part coding to be agreed (i.e. using some order and some form of classification), then checked for ease of use and consistency.

This task would equally well in the school grounds as a practical exercise and encourages language abilities and wider thinking about space and the description of space.

Sharing: The whole class looks at solutions from different groups. Responses are likely to include:

- Giving directions from the house. Perhaps using compass direction like North West, then counting trees.
- Splitting the 'map' into quadrants, then using top-left, top-right.

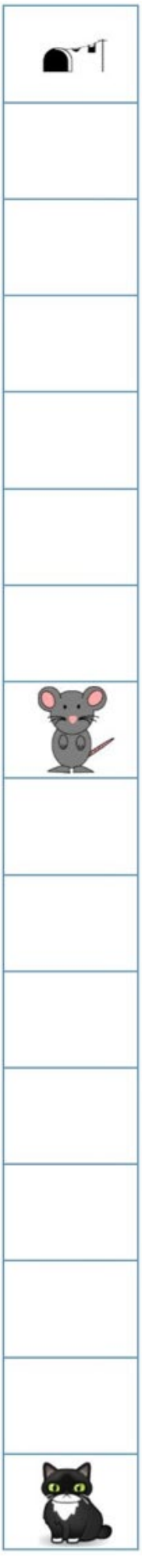
The joy of open-ended tasks like this one is that they can go in any direction, to suit the needs of the individual, class and curriculum, and provide a good hook to hang ideas on. A similarly open-ended and engaging task was suggested by Mark Pepper in *Struggle* Vol. 29 in 1991.

Cat and mouse game – From Mark Pepper

'What are the chances?' Hounslow Teacher's Centre Maths Advisory Team

Struggle Vol.29 1991

A game for two, using the grid in Figure 3, who can discuss what they think the outcome will be. The players take it in turn rolling a 1-6 die. Regardless



of which player rolls the die, if a number between 1 and 4, inclusive, is obtained the mouse moves that many spaces whilst if a 5 or 6 is obtained the cat moves that many spaces.

The mouse has to get to the hole before the cat gets the mouse.

Unfortunately, there is no quick fix solution to identify every individual's personal barriers, and no simple solution to teaching mathematics in a way that is engaging and accessible to all. There are, however, some key principles that are worth considering:

- Know your learners – what they can do; what they think they can do; what their mindset is; how they perform in other subjects; what gives them confidence and what damages their confidence.

- Make sure that they have the skills required to move on – getting 50% in an end of topic test is not mastery, and most learners forget what they learnt in year 8 by the time they get to year 10. Keep going back and revising core skills and discuss understanding of numerosity whenever you can.

- Use concrete examples to help develop understanding and provide engaging ways of teaching a topic. Allow learners to make sense of these ideas using pictures and diagrams as well as words and algorithms.

- Mental arithmetic and times tables are useful skills and make using number more efficient, but they can also set barriers that can lead to greater difficulties. Pressure of answering quickly and being asked to present in front of the whole group are not approaches that suit everyone.

Finally, keep reading *Equals Online*, and if you have a lesson idea or approach to learning that you think is worth sharing get in touch with the editor. The more people that share ideas, the better *Equals* becomes.

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2. Chinn, S. (ed) 2015. *The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties*. Particularly chapters by Desoete, Landerl and Emerson
3. Sharma (2012) http://mathematicsforall.org/evaluation%20process%202012.pdf

Finding the Frog!

In this article Alan Edmiston recounts a lesson he recently taught at Charlton Park Academy, Greenwich. The hope is that readers will use the resources to try the activity with their SEND pupils and then to send in their thoughts and reflections. Information about the nature of this special school can be found at the end of the piece.

As part of a Key Stage 1 Let's Think Math's course organised by the Compass Partnership of Schools in Greenwich I had the pleasure of visiting Charlton Park Academy. A large part of the course involves the active teaching of lessons as part of the planning process and during this day I tried a lesson on co-ordinates with a small class of Year 10 boys. The rest of the group were then able to teach the same lesson later in the day. In this way everyone gets the chance to both observe and teach a lesson before taking it back for use in their own schools.

I chose the **Finding the Frogs** lesson as we had decided to look at a series of activities that come under the umbrella of Shape and Space. Despite not having taught it with pupils of this age I felt confident we could have some fun. I must have taught this activity 30 times or more and usually I try it with pupils in Year 2 or 3. Much of the lesson involves the pupils playing a game that challenges the fact that positional language i.e. left, right, top, middle, corner, on top of etc, does not help to quantify space in the way that co-ordinates do. Given the pupils were working at level 2 I was interested to see how they would make the move to quantifying space and even if the lesson would serve that purpose.

I will outline the lesson, which has two main parts, and the resources before sharing what happened when we tried it.

Episode 1 – finding the frog

The first part of the lesson involves the pupils playing a barrier game where each player is unable to see the other's sheet. The aim is for player 1 to place the four objects in the pond and then to describe (as if on the phone) where they are so that player 2 can place them in the same place. They can then swap roles as many times as required. In Year 1 classes this can take some time. This episode is linked to the story of a frog who lives in a pond and the objects she encounters each day.

The aim is to develop a language to describe where the objects are in the pond so that each can be

correctly placed when described. This aspect is usually stimulated when, after the first round, both ponds are placed side by side and the mistakes can be clearly seen. Much discussion then results as each player thinks the other is the cause of the mistaken instructions.

This part of the lesson ends with time of reflection where all agree on the best way to give instructions. There is also a consensus as to the difficulty of describing an object when it is not next to sometime, or in the middle or a corner. At this point a discussion takes place where the sheet used in episode 2 is developed with the pupils.

Episode 2 – developing co-ordinates to help position the frog

The sheet above is used to enable the pupils to practise placing the frog, after she has jumped, into the pond but only following a time of whole class discussion that approaches the quantification of space. This time is a bridge between the problem facing the pupils at the end of episode 1 and the more formal use of co-ordinates at the end of episode 2 and is designed to enable the pupils to visualize the way we can divide up space.

A blank pond sheet is used, the larger the better, and the pupils are asked to imagine the frog living in the bush and waking up every morning and hopping along to the right before deciding how far to jump into the pond. In the manner of 'The Very Hungry Caterpillar' on Monday the frog wakes up and makes three hops along before deciding to make two hops into the pond. If it is stressed that the hops are the same size each time and then on Tuesday she hops along four before jumping two in. If this is done another time the pupils can begin to see how the two can form a visual line across the pond. This can be done a few more times so that a whole series of imaginary, numbered, lines are drawn both horizontally and vertically in the pond. Only now can they move on by following your lead placing the frog in the pond using the language of co-ordinates i.e. 4, 2 or 0, 3 etc.

"I bet we can trick you Sir!"

The boys at Charlton Park responded brilliantly to the idea of the game and really engaged with episode 1. They took great care to describe the position as they placed each object and yes they were surprised that mistakes were made. I think they played the game for about 20 minutes before one of them, John, started to use the idea of half-way to get round the fact that they did not really have a language that enabled them to quantify space. Most played safe and put the objects in positions they could easily describe them. I sat next to this boy and his partner and suggested that I could trick him into making mistakes. With his agreement I placed the objects for him to describe for his partner. I took great care to place the objects in positions that were hard to describe. The look on his face

was wonderful but he bravely tried to describe the objects to his partner. 'That's not fair Sir" was his cry and one which focused the class on what we were doing. He was then able to describe what I had done to trick him.

They then asked if they could all try and trick me in the same way. I agreed and left the class for five minutes while they placed the objects in 'tricky' positions in the pond. Upon my return they sat me behind a screen at the far side of the room while all ten of them sat with the adults at the other. It was John who was given the task of describing the objects for me to place. He realised how difficult a task this was and in response began to say half way and half again to describe the position of the flower. As he did this, and unseen by the boys, I began to mark my sheet by folding it in half and half again, both vertically and horizontally. In this way the 'lines' were clear to see and I was then able to place each object in the correct place.

To finish I held up my sheet and asked them to describe what I had done to place the objects and if they could imagine dividing up a space into ½'s and ¼'s in this way. I finished by drawing in the folds with pen and asking if they could be numbered. We were then able to number the vertical and horizontal lines. I am sure that if time had permitted we would have progressed even further through episode 2.

My time at Charlton Park will live long in my mind for two main reasons;

- Firstly the engagement and enjoyment of the pupils was a pleasure to behold and as a consequence their ability to reflect upon and articulate their thinking was very humbling.
- Secondly the activity itself was able to bridge a gap in their thinking and move the pupils on.

This is lovely to see in an age when far too many pupils are rushed and then simply told how to do it. Mathematics is and has always been a human endeavour - I think we do our pupils, whoever they are, a dis-service if we ignore this.

If you would like to try this article please email edmiston01@btinternet.com for copies of the worksheets used.

If any reader would like to visit Charlton Park Academy please get in touch with Alan Edmiston: edmiston01@btinternet.com
On the 26th September they are having a one-day conference to share how they have used the Let's Think materials with their pupils.

Information about Charlton Park Academy

This is a residential special school that is maintained by the Royal London Borough of Greenwich. It provides 220 places for students aged 11 to 19yrs who have a statement of special educational needs. Student's needs include Autism, complex and /or severe learning difficulties.

Charlton Park Academy also provides eight residential places, in four shared bedrooms in the on-site residential unit known as Rainbow House. There are currently 197 students on roll, 12 of whom use the residential service, though two of these do not stay overnight. Rainbow House is situated in the main school. The residential service is available during week days and term time only.

Reflections on a weekend in Surrey

Pete Jarrett **kindly represented *Equals* at the recent Easter Conference. His thoughts on this experience highlight what a rewarding experience it was and how much he was able to take away.**

The editorial board of *Equals* Online have decided that we need to get out a bit more! This year it has been my privilege to attend the MA conference and give a presentation on an aspect of SEND and mathematics. Alongside this presentation, which is included as a paper in this issue of *Equals*, I wandered around the Conference picking up a sense of how our profession is viewing mathematics teaching and learning, as well as taking part in the occasional quiz and wandering around the grounds of Royal Holloway University with Florence Nightingale!

I spend quite a lot of time at Conferences, most of which are more SEND than mathematics focussed. Because I am at these conferences to talk about mathematics learning difficulties I tend to meet a wide range of practitioners; some who are mathematics teachers, others SENCo's, senior leaders and teaching assistants. No Conference ever goes by without an "I'm really bad at mathematics" or "my mathematics teacher hated me". The best I have heard this Spring was from a Head Teacher who told me the "I was put off mathematics when I was around 8 or 9. My teacher threw the board rubber at me!". Yep, that'll do it! This has always confused me as I have always found maths teachers to be pretty amiable, and whilst the physics of an airborne board rubber are intriguing, most teachers want all their students to do well and enjoy the subject, rather than cower from flying objects and develop a lifelong loathing for mathematics.

We have always reflected on what makes mathematics engaging, enjoyable and accessible, and, perhaps, we are most constrained by the rigours of the assessment system and countless measures of accountability than we are by a lack of desire to get our students engaged. But, we know that mathematics is not always an easy subject, that it can be hard to grasp and finding learning difficult can be frustrating. We know that every learner approaches mathematics in a different way, and makes sense of their learning and subject knowledge uniquely. We know, as teachers, that our default approach to explaining a task may present a challenge for learners who make sense of the subject in a different way. I had some interesting discussions at conference about differing approaches to problem solving, about the different ways that geometers and algebraists approach straightforward tasks, and other generalised approaches to thinking that exist amongst mathematicians. With such diversity amongst teachers and learners alike it is no surprise

We have always reflected on what makes mathematics engaging, enjoyable and accessible

that teaching for understanding for everyone is so difficult.

So, what themes emerged from this year's conference that may be of interest to teachers with an interest in SEND?

Firstly, there has been much talk about grit, resilience and mindset. This seems to come from two, equally valid, viewpoints; that learning is tied to emotions, and that learners do best when they believe they can improve and that mistakes are part of the learning process; and, that learners need to be willing and able to think deeply about problems to have the understanding to solve them. Positive psychology is a relatively recent visitor in the classroom, and, like the best ideas, seems to be common sense explained. We have recognised the issues around self-confidence and self-efficacy in learners with specific difficulties for a number of years. The late Professor Robert Burden published a great book, 'Dyslexia and Self-Concept' in 2005 which I have always found to be really useful in positioning my practice, and now, I feel, it is common practice to address issues of resilience and confidence in learners with SpLD's.

learning is tied to emotions, and learners do best when they believe they can improve and that mistakes are part of the learning process

In the mathematics classroom, I would argue that many of the best maths teachers have been doing this for years, and that we now recognise that we need to get learners in the best place to learn before we can start to teach. This particularly applies to learners who struggle.

The second key theme that I think is of relevance to readers of *Equals* is the use of concrete manipulatives to help learning in the classroom. We traditionally think of concrete manipulatives to be Numicon, Base Ten tools and Cuisenaire Rods. These have a place, and their stock is rising because of their use in the 'Singapore' approach to maths teaching and learning. In addition, conference was full of engaging real-world ideas that are useful for learners of all ages – paper folding was a real highlight for me, and if you can keep a room of mathematics teachers engaged, you can keep a group of students engaged!

The MA conference provides an excellent opportunity to share experiences with teachers and academics and to get a feel of what we collectively feel is helpful in the classroom. As both a mathematician and specialist teacher of SpLD's I have always felt that if we manage the affective domain to get learner in a place where they are ready to learn, and we teach them through real-world modelling alongside and leading up to more abstract understandings, then we will turn out learners who are less afraid of mathematics, more confident in their skills and abilities, and experience more of those joyous lightbulb moments when a concept clicks with a learner.

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Fun Activities from *Struggle*, *Equals* and *Equals Online*

What a rich resource *Equals* has become. In this article Mark Pepper looks back at almost twenty years worth of activities that teachers have used to make fun and enjoyment a key feature of their pupils mathematical experiences.

There appears to be a high correlation between the enjoyment of maths lessons by learners and their engagement with the subject. Hence it is beneficial to include interactive activities which are fun and can be of assistance in reinforcing mathematical skills that are being learned within a more formal process. Furthermore some fun activities can act as a starting point for greater conceptual understanding within mathematics. A further positive aspect can accrue in that some of the activities provide ideal opportunities for differentiation.

When I reflect upon the fun activities that I found to be popular with learners I am surprised to discover the very wide time scale that they embrace. All of the following activities have featured in *Struggle* (the forerunner of *Equals*), *Equals* or *Equals Online* with the earliest being published in *Struggle* in 1989 and the most recent in *Equals Online* in 2016. Hence it covers a time span of 27 years! I have used all of these activities intermittently throughout my teaching career.

***Number Spiders* (1989)**

For this activity each learner is provided with a sheet of paper with the outline of a spider drawn on it. The eight legs of the spider protrude from its body and at the end of each leg a box is drawn.

A target number is written in the body of the spider and the learner is then required to complete each box with a number sentence in which the answer of each statement corresponds with the target number.

This activity is particularly useful in a class in which there is a wide range of ability. Hence at a basic level the target number could be 10 and the learner could complete the boxes by inserting number bonds up to 10. At a more sophisticated level the target number could consist of 3 digits and a mixture of number operations could be used in the number sentences.

Number spiders provide sufficient flexibility to enable the teacher to provide number targets in accordance with the perceived level of ability of each learner.

***BMX Game* (1991)**

In this game all of the players use the same grid. The grid consists of 12 columns and the base of each column contains a number from 1-12 inclusive. Above these numbers are 10 rows. The

players choose numbers for their bikes in advance of playing the game. The game is played by each player in turn rolling two dice. The total is then calculated and the bike of that number makes one move forward. The eventual winner is the bike that reaches the finishing line first which is located in the top row. When I used this game with a mixed Years 3 and 4 class it generated plenty of excitement with a considerable amount of competitiveness as each player was willing her/his bike to win. After we had played the game on a number of occasions It rapidly became apparent to the players that it was not an arbitrary chance which bike would win but that bikes such as 6,7 and 8 regularly tended to do well. This led to a class discussion in which the players were encouraged to give reasons for the success of some bikes and the relative failure of others. It was then decided that we would find a way of calculating which bike number had the best chance of winning. As a whole class activity we recorded all the possible outcomes of adding the scores of two 1-6 dice. Of course this demonstrated that 7 had the most possibilities followed by 6 and 8. We then produced a bar chart of the results and the class were struck by the symmetry that was produced. Hence a considerable amount of mathematical learning took place from a starting point of playing a game.

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Cat and Mouse game **(1991)**

This is a game for two players on a grid used by both players. A picture of a cat is located at the bottom and this is followed upwards by eight spaces where a mouse is drawn which is followed by a further eight spaces and then the words “mouse hole”. The players take turns in rolling a 1-6 die. Regardless of which player rolls the die, if a number between 1 and 4, inclusive, is obtained the mouse moves that many spaces whilst if a 5 or 6 is obtained the cat moves that many spaces. If the mouse reaches the mouse hole before being caught up by the cat, the mouse wins. If the cat does catch up with the mouse then the cat eats the mouse and is the winner. When I have used this game with a class it has usually generated a lot of interest and much discussion has taken place on the expected winners with reasons to support the view that is taken.

Dice magic **(1994)**

This activity evolved from some work I was doing with a Year 3 class involving number operations with the use of dice. One of the issues that arose was the fact that the sum of the opposite faces of a die is always seven. In the course of a discussion about this it was agreed that if we looked at the value of the facing face then the value of the opposite face could be found by subtracting from 7. One of the girls then said that if we had two dice the sum of the two opposite faces could be calculated by subtracting from 14. She went on to say that the same principle would apply with three dice in which case a subtraction from 21 would be required. It was then suggested that we should make use of this information to perform a “magic trick” in a forthcoming school assembly.

It took place at the conclusion of an assembly. Three jumbo dice were placed on the stage in front of the audience. Two members of the class, who I will call Abdul and Janet, sat on the other side of the dice facing the audience. A member of the audience was invited on to the stage to rotate the dice. The audience was then told that Abdul and Janet would be able to tell them the sum of the faces that they could see but which could not be seen by Abdul and Janet. They then added the values of the faces that faced them and subtracted the total from 21 and correctly announced the value of the sum of the three faces that they could not see. The dice were rearranged a number of times and each time Janet and Abdul correctly identified the sum of the faces that were unseen by them. The audience were greatly impressed by this demonstration of magic!

Countdown **(1996)**

This activity is based on the numbers section of the television programme Countdown in which a 3 digit number is produced at random. Contestants are then allocated 6 numbers. Usually one of the numbers will be 25, 50, 75 or 100. Each of the remaining numbers will be between 1 and 10 inclusive. The contestants can then use the allocated numbers and any of the operations on the numbers to try to obtain the target number. They do not have to use all six numbers.

The audience were greatly impressed by this demonstration of magic!

A particular strength of this activity is that it can be differentiated according to the abilities of a particular class or within a class of mixed ability. In the television programme the numbers are randomly produced whereas in a classroom context the numbers can be selected in advance by the teacher. Hence with a Year 2 class an appropriate set of numbers could be:

Target 26 Numbers 10,2,3,2, 1

When working with Year 4 learners an appropriate selection could be:

Target 316 Numbers 75,2,2,3,5,1

A more challenging problem could be:

Target 617 Numbers 50,2,2,3,4,5

I used to find it helpful to have a bank of Countdown problems with varying degrees of difficulty available. They could then be used in the event of a learner finishing a task earlier than expected.

Usually learners showed considerable enjoyment in participating in Countdown activities.

Giveaway **(1997)**

This is a useful activity for pupils who are in the process of learning number bonds up to 10 or 20. This game can be played by approximately five players. At the start of the game each player receives ten cubes. They then take turns to roll a die and give the appropriate number of cubes to the next player. The game is concluded after three rounds and the winner is the player with the most cubes. Whilst the players usually enjoy the competitive nature of the game, the learning of number facts can be reinforced by posing questions such as:

"You started with 10 cubes. You gave 4 away. Count how many are left".

After an answer of 6 has been supplied the statement can be made:
4 add 6 = 10.

***Card Game* (1997)**

In this game four sets of cards are used. Each set contains four cards with the same numbers e.g. 6, 7, 8 and 9. All of the cards are amalgamated and shuffled thoroughly and the pack is then placed face down. Players then take turns in picking up the card at the top of the pile and saying how many more would make 20.
e.g . A player picks up an 8 and would then need to say "12".

If the answer is correct the player keeps the card whilst if the answer is incorrect the card goes to the bottom of the pile. The winner is the player with the most cards at the end of the game.

***Down the Pit* (2016)**

This is a game that helps in the learning of multiplication tables and factors. The teacher writes the numbers 1 to 20 on a flip chart and then draws a ring round three of the numbers that have been randomly selected. The learners then have to pick a low number that they think will miss the ringed numbers when they are continuously added. E.g. if the child chose 3 then the numbers obtained would be 3, 6, 9, 12, 15 and 18. It is helpful for the teacher to use a jumbo calculator with the display facing the class and slowly display the numbers generated by using the constant button. The teacher can keep a record on the flip chart with two columns – one headed will work with the other headed won't work. This helps the class to recognise patterns. E.g if a ringed number is 16 they will usually start to realise that 2 should not be chosen as 16 is an even number. This game can easily be adapted such that for Classes 3 or 4 the numbers could go up to 50. A further adaptation of the game can be used to reinforce the concept of prime numbers by the teacher drawing a ring round three prime numbers. The class will soon discover that these numbers have no factors except 1 and the numbers themselves.

This helps the class to recognise patterns

Mark Pepper

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Math's talk

Math's talk this time comes from a conversation with Joanne Thornton who is in charge of teaching and learning and assessment at Mortimer Community College in South Tyneside. Joanne has been a mathematics lead practitioner for many years and was first promoted to the role while working with autistic spectrum pupils.

Could you describe some of the key moments in your career?

One that stands out for me was working with autistic pupils at my previous school. To succeed I had to focus upon developing relationships with the pupils which then enabled me to develop their engagement with mathematics. To help with their social skills, as well as reinforcing mathematical skills, I used many Kagan structures, cooperative learning, with them. I find these ideas now underpin all of my teaching.

My own son's primary school had a real influence on me. Looking at his books I was struck by how clear and supportive the marking was. He was able to move forward using the next steps provided by his teacher. This prompted me to take a long and hard look at my own practice, and that of the whole school, and respond accordingly.

To succeed I had to focus upon developing relationships with the pupils which then enabled me to develop their engagement with mathematics

Who has been the biggest influence upon you?

That was Dave Pentland my first head of Department when I started teaching at Westgate Community College in Newcastle. His classroom practice stood out for me and was in stark contrast to the other classrooms I had visited. He always sought to engage his pupils and the lessons were always fun. He was very much into independent learning and differentiated support.

Mixed ability or setting?

I have worked with both and can see the advantages and disadvantages of both. I know setting has been found to have no impact on the students' learning but I now prefer to teach setted pupils as its easier to plan appropriate lessons and challenge them from their particular starting points and to also keep an eye on them so they don't slip through the net.

What are your thoughts on some of the recent developments in mathematics education?

Currently my thinking is taken up with my Masters work on engaging staff with CPD. This is helping me to think about how we really share good practice and encourage staff to take risks and share what

they are trying out in their individual classrooms. As a result I am thinking about introducing a walk and talk which is not the same as a learning walk with its associated labels i.e. WWW and EBI (What Went Well, Even Better If). My aim is to foster engaging conversations among all teaching staff about what is taking place and the practice they see.

When you observe a math's lesson what do you look for?

I always look to see if the pupils are being challenge by their teacher and that the materials that are being used, are engaging the class. Fun elements are important to me but this cannot always be the case. It is essential that all students are given the chance to succeed, that scaffolding is put in place to ensure they feel confident to attempt the work provided.

What advice would you give to anyone who wishes to support the lower ability pupils within their classroom?

I would encourage them to use fun activities (games are a good way of both engaging and motivating the students. Many shy away from this because of potential behaviour problems but in the long run it pays dividends. It is important to relate their learning to real life situations where ever possible as this allows them to see a purpose to the learning. I also would ask them to use lots of praise and encouragement but not in a false way – take the time to really celebrate what they are doing and their efforts.

take the time to really celebrate what they are doing and their efforts

Another important thing to do is to get to know where your class is at by using formative assessment. Once you know where are they starting from it is so much easier to respond to their specific needs and from that progress will come.