Equals

Realising potential in mathematics for all

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Remebering Rachel Gibbons

Realising

potential in mathematics

for all

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Designed by Nicole Lane

Editor's Page

From the picture on the front it is clear that this edition of *Equals* is different. Most of our articles serve as a tribute to Ray Gibbons, the driving force behind *Equals*, who died earlier this year. It was Ray whose passion for all learners gave birth to something called *Struggle*. It was her determination that saw *Struggle* grow and develop into *Equals* and so without her we would not exist.

The three pieces about Ray have been written by those of us who knew her best: Mary Clarke, Mundher Adhami and Mark Pepper. I did not have the pleasure of meeting Ray in her prime but each time I spoke to her I caught a glimpse of the woman who cared passionately that all children were born with the ability to think and reason mathematically. Three years ago it was poor eyesight that caused her to step down from running *Equals* and at that point I tentatively agreed to steer things for a while. I still feel in awe of what she did and it is only now that I feel *Equals* is becoming the force it once was. If you met Ray, or have any stories about her, then please get in touch.

In honor of Ray we are pleased to republish a piece written by her and also to announce the Ray Gibbons Memorial Award. This prize will be given to anyone you feel deserves recognition for their work within the field of SEND and mathematics. If you know someone who, you feel, needs to receive public praise for their work then please let us know. A short biography and a few words on why you are recommending them are all that is required.

The most significant development to report at this time was the invitation to *Equals* to help support the Math's Hubs SEND work groups. Pete Jarret was able to represent us at the first meeting in September and the most recent copies of *Equals* were shared with all those who came. As part of our involvement we are planning to use our Spring edition to focus upon this work and also to share the direction of SEND development as seen though the eyes of the Maths Hubs.

I would also like to introduce you to two new members of the *Equals* Editorial Team: Louise Needham and Kirsty Behan.

Louise Needham is the curriculum advisor for mathematics at the New Bridge Multi academy trust. The trust is a group of special schools in Oldham and Tameside that caters for children with all varying types of SEN from EYFS to college. Louise is very passionate in making mathematics accessible and enjoyable for all.

Since graduating from LSE with a degree in maths and economics, Kirsty has worked in comprehensive secondary schools in North London. After working as a teaching assistant for several years alongside studying her masters in education, with a focus on maths and inclusive education, she qualified as a secondary maths teacher. She is passionate about supporting the increasingly complex students, and their teachers, in mainstream education to engage with and succeed in mathematics.

Take 4 Triangular Tiles Special Educational Needs - what are they? and how do we meet them?

Rachel Gibbons **follows up some of the questions posed at the *Equals* presentation at Special Needs London last September. She suggests that before looking at How to teach we should explore the Why and What questions presented by our pupils. She explores an activity using 4 triangles to illustrate the point.**

We promised to follow up queries that arose at our Special Needs London presentation last autumn. Preparing for that session made us revisit the aims of *Equals*. Reflection of this sort is an essential part of the team's continuing professional development, and something that all teachers should continually be engaged in. Our education should be continuous, a life-long enquiry, continuing those earliest questions framed by the infant:

Why …? What …? How …?

The questions we were asked at Special Needs London were mostly the how questions:

How do you teach X to Y?

But I would suggest that is the final question, not to be tackled until after you have answered what? and why?:

What are the special needs of the children in front of us? and

Why should they learn topic Y, Z or W?

The **how** depends on the answers you give to the what and the why and also on the techniques and resources you have at your disposal. HMI when assessing a mathematics education project back in the 80s commented:

> 'The best teachers had a **detailed knowledge of**
> - **Mathematics;**
> - **the material;**
> - **and the children;**
>
> they used this knowledge, together with sound judgement and some initiative, to select appropriate and valuable tasks for their pupils.'

We feel that one question listed in our participants' lists: 'How do you teach multiplication to dyslexics?' has no sensible answer. The comments above suggest that a more appropriate series of considerations would be:

- Firstly, consider the child I have in front of me who, amongst a multiplicity of other characteristics, has been described as dyslexic - what are her capabilities, interests, expectations and achievements to date?
- Secondly, consider the materials that are available including not only the equipment, books etc., but your own understanding of the subject and the child's classmates;

- Thirdly, comes the mathematics – why mathematics? Which mathematics will be of interest/use to this particular child?

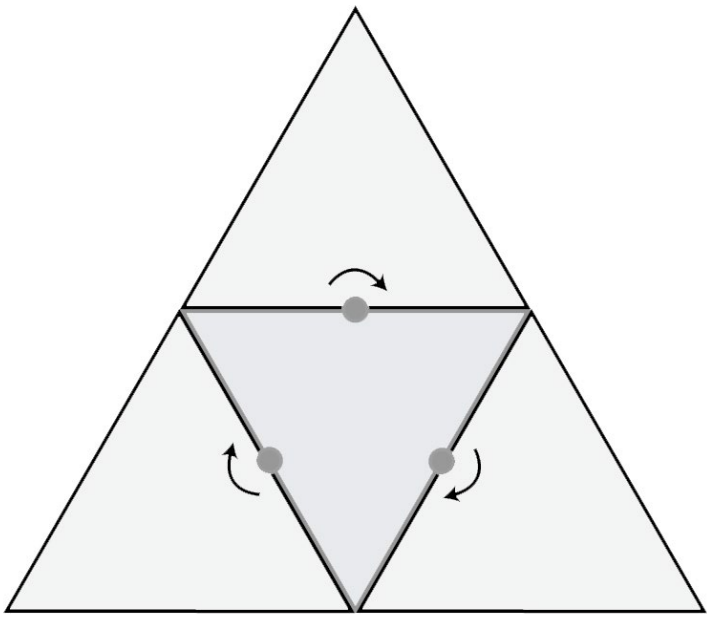
Use the national curriculum as a guide; it has been put together, after all, from earlier proven schemes by people with long years of experience in teaching the subject. But, because it is national, and therefore the lowest common denominator, adaptations have to be made for the particular set of insights, stumbling blocks, etc. found in each child. This is why in *Equals* we aim for a good proportion of articles about general good practice - effective ways of putting mathematics across to anyone anywhere. If children have extra difficulties outside the norm then they need that and much more. In other words, to teach children with special needs effectively you have to be an extra-special teacher. A teacher in a special school told me recently that someone had asked her why she had to have extra training to teach in a school of that sort. Their argument went that because the children she was teaching would not learn as much as other children, then surely, she would not have to know as much as a mainstream teacher. We would argue, however – as she did - that one has to have a clearer grasp of what one knows, a deeper understanding, both of the subject in hand and of the problems being encountered. This is necessary in order to see different ways of getting concepts across to children who have difficulties with learning.

to teach children with special needs effectively you have to be an extra-special teacher

To take an example, if a child has any kind of visual impairment, your presentation of geometry will have to change because the usual approaches involve **looking at** lines, curves, triangles, cuboids, and so on. I once had, in a fully-sighted mainstream Year 11 class, one boy who had very little sight. The advice I was given by a senior colleague - whose views on education I usually respected - was to treat him like the rest of the class. Well, of course, I could have. But, if I had done, what would he have learned? How does one look at shapes if one has only partial or no sight? And what is one aiming to learn about them in mathematics lessons?

Perhaps it is worth looking at that triangle in more detail because one area of mathematics reported as causing difficulty was 2D and 3D geometry. A lot of experience through the senses does help to form the abstract concepts of geometry and it is useful in the example we are considering to see what the available equipment has to offer in the way of triangles that might help those who have sight problems. Pencil, paper and ruler are of course the first and most universally available materials. Drawing gives physical experience of straight lines and turning though angles of different sizes. If scissors are also handy then you can cut out triangles and move them around and turn them over. Then you can make a set of copies and do some tiling, seeing how they fit together. At this point the child with visual impairment can come in because he can handle triangles that are cut out. Is my set of triangles the same shape/size as yours? My set of triangles fit together exactly, does yours? And yours? And yours? … in other words, comparing experiences around the class is invaluable. The cut out triangles will be of use to the child who is partially sighted because his seeing will be through his fingers. For children who prefer to learn through kinaesthetic experiences this approach will also be beneficial. It is useful to gather together sets of tiles, maybe shapes cut out of materials with different surfaces which can be distinguished by touch. The equipment about which Jane Gabb writes (also following up on the questionnaire responses) elsewhere in *Equals* 10.2 should prove useful to those with defective sight. We also need plenty of examples of 3D shapes. Perhaps we should be starting geometry for all with 3D shapes, not only for those who must handle rather than look; after all, we live in a three dimensional world.

Back to our triangles– let's take 4 triangular tiles ready-made or cut out of cardboard, stiff card, vinyl or whatever is available. Let's begin to rotate these tiles through a half turn (180º) about the midpoints of each of their sides. To do this the non-sighted students could do with a stack of 4 tiles, 3 of which can be lifted off the pile and rotated into positions next to the original with matching sides touching as shown below:



The non-sighted student can now 'see' the triangles with their fingers, finding out

- what shape combining any one of these repeats with the original has made,
- what shape this combination of the original and its 3 repeats have made,
- what this tells us about the angle sum,
- whether this makes sense in the light of how they have been moved.

And what more can be found out about the relationships between shapes made by combining the four tiles? There are parallelograms to be found and similar triangles, there are sizes to compare – lengths and areas - and … I am sure you can think of much more. For, surely, education is a voyage of exciting exploration rather than the cold transmission of sets of rules.

Perhaps you will complain that this does not begin to give enough specific answers to the 'how' question but maybe it takes us further with the 'what' and the 'why'. This can make us more generally effective teachers with a deeper understanding of a piece of mathematics, through a simple activity (for the teacher learns too when shapes are manipulated). This can give us a fresh view of some of the materials at our disposal, and a more detailed knowledge of the particular difficulties of the child in front of us. Then we will be better equipped for our very exacting (but always exciting) task than we were last time round.

Which brings us to the consideration of some of the other questions asked in the questionnaire responses:

- How do you provide fun and inspiration?

- How do you engage them with mathematics?
- How does one move from concrete to abstract?

And this is where the teacher comes in. The excitement, the inspiration and the interest in abstraction must come first and foremost from **you**. Watching David Attenborough exploring the story of amber recently I was enthralled. Why? Well, I suppose I found it a fascinating subject to begin with. But what made it really inspiring was Attenborough's own excitement. So maybe the final questions for every teacher (whatever the subject in hand, whatever the characteristics and achievements of the children in the class) should be:

- How do **I** find fun and inspiration in this activity/exploration?
- How do **I** engage myself meaningfully in mathematics (or any other part of the curriculum)?
- How do **I** move from concrete to abstract?

And here is the rub. So many of us in the classrooms of today have been taught mathematics so badly ourselves that we have failed to catch the inspiration it has to offer. How can we catch the excitement? Perhaps these are questions we can explore further in a later issue of *Equals*.

Ray Gibbons Memorial Award

This prize will be given to anyone you feel deserves recognition for their work within the field of SEND and mathematics.

If you know someone who needs to receive public praise for their work then please let us know.

A short biography and a few words on why you are recommending them are all that is required.

Get in touch: equals@m-a.org.uk

Thanks to Rachel Gibbons

Mundher Adhami, **Mary Clarke** and **Mark Pepper** pay tribute to their friend, and the founder of *Equals*, **Ray Gibbons**.

Ray Gibbons, my mentor, and a champion of lower achievers

This past July, we lost Ray Gibbons, who had given most of her 90 years to mathematics education. A short but warm informative obituary by her friend John Hibbs was published in the Guardian https://www.theguardian.com/science/2018/aug/07/rachel-gibbons-obituary. John is a prominent Mathematics educator at the HMI, Open University and the Association of Teachers of Mathematics, who had worked with Ray from the 1970s on the SMILE project, and shared much of her beliefs. His obituary covers parts of Ray's private and professional life as inspector in the Inner London Education Authority, (ILEA), that remarkable incubator for progressive educational ideas and projects.

I too have known Ray since the 1970s, when I started teaching at Elliott School in Putney, London. I have always known her as a mentor, in the sense of a supportive and responsive expert colleague. Her style of mentoring in line with her own strong views but also with seeking other views with respect and support for initiatives. This is a role common amongst educationists, coming more naturally to some than in others. It came naturally to Ray Gibbons till late in her life, up to when she reluctantly let-go of editing *Equals*.

Perhaps expanding on his mentoring role would be a fitting tribute to Ray, as well as a way of acknowledging in passing the support I had received from many colleagues who are now mostly of greater age than mine.

Mentoring as a natural aspect of teaching

Ray was one of the many mentors I had in succession, and she figured sporadically at different times throughout my 40 years in teaching. Starting with me at Elliott School in Putney relying upon the Secondary Maths Individualised Learning through Experience (SMILE), the scheme Ray had initiated. Then while being a head of department at Wandsworth Boy's School then Head of Faculty of Mathematics and Technology of John Archer School, the outcome of an amalgamation of two departments and two boys secondary schools. Then, when Ray recruited me to jointly edit the Maths Association magazine *Equals*, a role which was later undertaken by Jane Gabb, till recently an advisor in Windsor and Maidenhead. In between these ‘stages’ there were times we met at ATM and MA conferences and even jointly ran workshops. Throughout this Ray and I had time for dialogue in pairs or discussions in wider groups, occasionally rigorous, on wider issues of educational philosophy and policy, which we both seem to have thoroughly enjoyed.

SMILE had ‘compassion’ towards non-specialist secondary teachers at a period of scarcity of maths teachers

I suspect this is the kind of life-long relationship with shades of mentoring most people in education have, with unrecorded formative effects on one’s views and perspectives. In my early professional life in teaching I had similar experiences with colleagues like Alan Jackson, head of Maths at Elliott School, and David Gibson, deputy head at Wandsworth and John Archer schools. Later on, I had support on occasion from many colleagues like Ann Watson and Malcolm Swan. To a different level of engagement than with Ray Gibbons, I can list “collaboration with mentoring” by Margaret Brown, David Johnson and especially by Michael Shayer, which is ongoing. ‘Mutual mentoring’ in the sense of supportive sharing of expertise with genuine respect for personal initiative is a feature of most research and development work and in charities like the Lets Think Forum with a great cast of trustees and practitioners in Maths, Science and English, some nearly half my age! I list these form of mentoring both as acknowledgement of formative effects of past and current colleagues and as a context for Ray Gibbons’ role.

SMILE and its Lesson Study model

SMILE was hugely important for me and I suggest for many teachers with no prior experience in active classroom learning, i.e. learning that is not reliant on someone lecturing and demonstrating at the board, or learners working through textbooks. SMILE relies on a large number of tasks organised by topic and level of difficulty. Each pupil has their own individual list of tasks appropriate to them for the fortnight. They start each lesson in the classroom by picking up resources, e.g. wooden blocks and worksheets, for the self-contained topic, sometimes presented in a game. Pupils work individually or in groups with the teacher helping as needed. They check their own answers and the teacher oversees the assessment and sets their next set of tasks according to a large sprawling grid organised by topics and levels of difficulty.

Ray’s philosophy of learning respected the individual learner’s mind’s own mysterious ways of fitting together fragmentary experiences in a Gestalt manner. It is a way to enable learning by discovery, while mistrusting rote learning and reliance on dreary textbooks (before the SMP, GAIM and investigations).

SMILE had ‘compassion’ towards non-specialist secondary teachers at a period of scarcity of maths teachers, by highlighting the role of teaching as ‘management of learning’ rather than pretending to be the font of mathematical knowledge and skills. That allows teachers with no great mathematics qualification, to engage themselves with the subjects, learning alongside the pupils. Often in my opinion this ensures causing less damage to children than in whole-class teaching, relying on poorly assimilated materials inevitably matched to a narrow range of skills. Inadvisable whole-class teaching would generally mean intervening in the learning processes by many pupils, and since you do not know what you are intervening in, may “do more harm than good,” in the words of Richard Skemp.

I think the process of producing the SMILE resources was as important for teachers' development as their use in the classroom was for pupils. Given the paucity in the 1960s and 1970s in teacher training courses, especially in psychology, sociology and philosophy, here was an empirical, integrated, hands-on approach in teacher training. It is a collectivist and democratic open approach where the best practitioners work with novices or less experienced colleagues in a ‘mixed-ability setting’. In easy-going workshops over long weekends or whole days funded by ILEA, lessons deemed by some teachers to be successful are pondered upon, split apart or elaborated to fit in a hierarchy of difficulty by voting and arguments, with the option always present for a subsequent change of collective mind. I suggest it is an early form of the Japanese Lesson Study approach, with neither authority nor formalities involved.

I now think the model of working in the SMILE teachers' workshop is superior to what we ended up using

The early SMILE tasks were targeted mostly towards the lower achieving pupils. Over the years a hierarchy of several levels (I recall 10 in my time) for the difficulty in knowledge and skills emerged, ending eventually with a GCSE syllabus based partly on moderated coursework. The striving for such a hierarchy in concepts and skills from the lowest to the highest in the school was more powerful then than now, when it is sadly neglected or frowned upon. Progression was perused by many projects including Kent Mathematics Project (KMP) and Secondary Mathematics Project (SMP). The notion of progression through a hierarchy of concepts in topics is implicit, i.e. without the explicit description of the level across topics, which later emerged in Graded Assessment in Mathematics, GAIM, then transformed and mutilated in the National Curriculum in 1989. Dylan Wiliam, now a renowned educationist, was then a Head of Maths in a London school and a major player in SMILE's resource development with Ray Gibbons, and later in GAIM with Margaret Brown. I had the benefit of working with these colleagues in SMILE and GAIM, and have no doubt that this experience fed into all my subsequent work in CAME, Cognitive Acceleration in Mathematics Education.

Reflecting on the way the hierarchy of levels in Secondary Mathematics learning was used and abused, I now think the model of working in the SMILE teachers' workshop is superior to what we ended up using, i.e. with heavier top-down theory-laden input by presumed experts. A messier, more open, empirical approach is probably less efficient, but probably also more empowering and real for teachers. Teachers need that messiness and experimentation more than refined products. Perhaps it is time we go back to open-workshop model initiated by Ray Gibbons and her colleagues at ILEA, of whom I recall two chief inspectors: Hugh Buxton, who became my employer in 1974, and Hugh Neil who seconded me to Kings College a dozen years later to work for GAIM, and later edited the first edition of CAME.

On leadership

I greatly valued Ray Gibbons support for me as a HoD. Given her left-leaning politics, she would have found an immigrant in such a position encouraging and exciting. I must admit that as an Arab 'over-stayer' at the time, I was not aware of any racism or discrimination in this country, although I had retained my Iraqi passport (withdrawn by the embassy in 1970) and only applied for a British citizenship two years after my second child was born in 1985. Perhaps I can ascribe this unawareness of racism to the support and mentoring by Ray and so many colleagues throughout my professional life, and to the success of ILEA's multiculturalism and to comprehensive education in London in general.

Ray's style of leadership was indirect and inquisitive.

Ray's style of leadership was indirect and inquisitive. She would enquire of you as a HoD the options as you see them, and then tentatively favour one, with mostly imperceptible added nuances. That did not prevent her from being ruthless when matters concerned pupils, as was the case with a teacher I could not handle myself, who was not really fit for teaching through rigidity and constant conflict about trivia with pupils. She found a way, and I don't know how, of sending him away, at a time this was very difficult, compounded by the difficulty of her being very strong union supporter.

I owe to Ray my having an Open University Diploma for Heads of Mathematics Departments, given after a 40 day course organised jointly with the Mathematics Association. This involved ILEA funding release of a dozen HoDs to work at Abbey Wood Centre with a brilliant academic tutor. The structure was a mixture of Master level work with practical workshops and written report on an innovative curriculum resource you develop yourself. We were paired, visited each other in schools, giving feedback on taught-lessons by ourselves, and forced to give joint presentations. That course felt more intense or useful than the MA in mathematics education at Kings which I did later, based, as it must, on up-to-date academic papers, essays and a longish dissertation. I hope such a practical course for new HoDs is still being offered under whatever title and funding available in the current climate. I recall though, that there was no significant structured input in that course on admin and management issues. The 1980s were still free of managerialism, and educationalists like Ray Gibbons were not alert to the coming onslaught, so were not yet armed with more suitable models for effectiveness than targets, value-for money, schedules and tick- forms.

"Struggle"* and *"Equals"

Ray Gibbons, like many public educationists, considered her priority the lifting of the standards of the lower half of the educational spectrum of achievement, often the outcome of social deprivation. In this view the high achievers can look after themselves with the resources all the schools offer, and are well catered for, and in fact 'if you raise the bottom, the whole column will automatically rise' through the dominant human need for comparison and differentiation. Additionally, recognising the many steps and difficulties in understanding lower level mathematics concepts is actually of great value for higher-level concepts, especially in avoiding misconceptions. This was the lesson of CSMS of the late 1970s (Concepts in Secondary Mathematics and Science) the spring board for GAIM, CAME and CASE

Ray convinced the Mathematics Association, normally engaged in advanced level mathematics and excellence, to produce a journal for the teachers of low achievers in Mathematics under the name *Struggle*

Ray convinced the Mathematics Association, normally engaged in advanced level mathematics and excellence, to produce a journal for the teachers of low achievers in Mathematics under the name *Struggle*. This was to emphasise the need by teachers to recognise the inherent difficulties in Mathematics for children, and the need to offer help accordingly, largely in constructivist or action based ways. Later on the title of the journal was changed to *Equals*. It was for many years a printed journal to subscribers. This is now a free online journal in its 23rd year, with three issues each year.

I had the privilege of joining the editorial group of *Equals* for a few years, recruited by Ray after editing a series of textbooks (Direct Maths) based on GAIM. We had editorial meetings at the Institute of Education and elsewhere and attempted to emulate refereed journals as far as possible. The editing of *Equals* involves at times helping practitioners to write their own experiences, and manipulating texts of various styles to suit what we saw as relevant to our audience of teachers of the lower half of achievement spectrum. Relevance often means seeing something that is useable in the classroom, but also stories of real interactions that focuses on what children think, their misconception and what insight they may have while handling difficult for them tasks.

The 2018 summer issue of volume 23 issue of *Equals* is here:
https://www.m-a.org.uk/resources/Equals-23-2.pdf

Ray kept control of the editing process in *Equals* even as she became increasingly frail and found travelling hard. She, at times, felt the MA was not sufficiently caring for the work and was a tough fighter for the cause. My experience from the MA council was of great respect for her views and fighting spirit, but also attempts to square the circle with finances, practicalities and contingencies. Some of us in the editorial group felt overwhelmed at times and wondered when in one's old age one should let go and let younger lot deal with set-ups they are more in tune with. At the same time admire the "not going gently into that good night", which means accommodating as far as we can what marvellous people like Ray Gibbons offer, well into dusk.

Mundher Adhami

Rachel Gibbons who died in July, aged 90, spent much of her working life in posts in the now disbanded Inner London Education Authority (ILEA), striving to improve mathematics teaching in the capital and in the UK more widely, not least through her membership and service in the Mathematical Association (MA).

Ray, as she was widely known, was born near Bristol and taught mathematics in Orme Girls School, Newcastle under Lyme, and Bath High School before moving to London. Her long career in the service of the Inner London Education Authority (ILEA) as head of mathematics at the Jewish Free School in North London, as deputy warden at ILEA's Ladbroke Mathematics Teachers' Centre, and as an ILEA inspector, gave her many opportunities to work with colleagues teaching mathematics.

Her enthusiasm, determination and strong principles were drivers for her ability to both challenge and support the thinking of teachers

She dedicated much time and energy to the ILEA supported School Mathematics Individualised Learning Experiment (later renamed School Mathematics Independent Learning Experience) (SMILE). Later after the abolition of the ILEA, Ray was instrumental in ensuring that the SMILE resources remained available to all via the STEM centre website. She was particularly interested in making mathematics accessible to children who have more difficulty in learning and in order to support teachers working with such children, for over 40 years, she edited the ILEA publication Struggle, later renamed as Equals and published by the MA after the abolition of ILEA.

As a passionate believer in the rights of all learners to have access to mathematics, she was a lead in promoting educational developments in mathematics teaching and learning. Her enthusiasm, determination and strong principles were drivers for her ability to both challenge and support the thinking of teachers so that they were enabled to develop lessons to motivate and improve mathematics learning.

Ray saw the MA as an important vehicle for the improvement of mathematics teaching in schools and she was a member from her first year of teaching until she died – 66 years in total. Her activities in the MA were multiple. Her articles were published in Mathematics in Schools; she ran workshops at MA conferences, sat on the Teaching committee and subcommittees (such as the Calculators and Interface subcommittees), as well as serving on the working party and committee for the Diploma in the Teaching of Mathematics to Low Attainers in Secondary School for 10 years.

All in all, a remarkable woman!

Mary B. J. Clark

I first met Ray Gibbons when I joined the editorial team of Struggle in 1988. I have always been greatly impressed by her determination to improve the learning of maths by lower attainers. Although she was a gifted mathematician, her interest was not in getting involved with one of the more elitist maths journals specialising in high status maths. Instead she became editor of *Struggle*, a magazine that specialised in maths for lower attainers. She was an extremely efficient editor and she also contributed many articles and reviews to *Struggle* and later to *Equals*.

In many ways it is not surprising that she was so committed to providing for lower attainers in maths as in more general matters she always supported the disadvantaged sections of the community. She was a strong opponent of right-wing government policies, especially those that involved education in general and maths in particular.

Ray's selflessness was remarkable. I recall one occasion at a *Struggle* editorial meeting when we considered an article written by a student teacher. Whilst the article made some interesting points it was poorly written in terms of use of language and correct grammar. Ray offered to work on it and in due course she rewrote it in its entirety such that all of the main points were retained and it was well written and flowed effortlessly. The original writer's name was appended in the published article but Ray did not include her own name.

When it was decided that the title of *Struggle* should be changed it was Ray who suggested that it should be renamed *Equals*. This was most fitting as Ray has always been committed to equality in all aspects of life.

Mark Pepper

The Renaissance of Cuisenaire Rods and Dienes MAB – Is this a positive development?

Mark Pepper takes the long view as he reflects upon the use of some old friends within the mathematics classroom.

Introduction

Upon reflection of Send Saturday, two significant issues come to mind. One consists of a strong consensus in the group in favour of the widespread use of Cuisenaire Rods and Dienes Multibase Arithmetic Blocks (MAB). The second involved a question from one of the participants which I will paraphrase as follows:

In a situation in which the learners are effectively using concrete materials, how does the teacher move them on to a conceptual understanding of the topic?

There is a direct link between these two issues and this will be explored in this article.

Current popularity of Cuisenaire Rods

The enthusiasm in the group for Cuisenaire is understandable, due to the current high profile of this resource as may be seen by the prominent place that it takes on a considerable number of educational web-sites. The ATM web-site in particular strongly endorses the use of Cuisenaire Rods. The longevity of the appeal of this resource is remarkable as its inception took place in the late sixties, approximately half a century ago!

The strong support of Dienes MAB within the group is perhaps more surprising seeing as this resource does not appear to currently have a high profile though it is still available on the internet especially on sites that provide used resources.

Personal perspective on the use of structured apparatus of "New Maths"

As I am of sufficient age to have direct experience of "New Maths" in the years immediately succeeding its introduction, the following will be written from a personal perspective and convey my experiences and attitude towards the use of these resources.

I attended teacher training college from 1973-1977 in order to train to become a primary teacher. Whilst my main subject was English, other key subjects were also studied. The training provided within Maths consisted of an impressive range of topics that were covered in considerable depth. The lecturers were strongly committed to the "New Maths" approach including the use of the structured apparatus that was an integral component of it. Within number the main resources consisted of Dienes MAB, Stern Apparatus and Cuisenaire Rods. The main resource within shape and space was Logiblocs (subsequently re-named Attribute Blocks). Whilst Stern Apparatus is still available it does not figure prominently on most mathematical web-sites and so it seems reasonable to conclude that it is largely in decline. Similarly Logiblocs, when first introduced, were an extremely useful resource particularly in teaching set theory which was then a prominent part of the maths curriculum. They were also a useful resource for reinforcing the identity of 2D shape as well as stimulating discussion upon their main properties. They have now largely been superseded by more recently introduced 2D resources.

The theory was that the children would gain experience of working in a variety of different bases through the use of the blocks.

The two resources that will now be considered more fully are Dienes MAB and Cuisenaire Rods.

Dienes MAB

At the time that Dienes MAB was introduced there was great optimism that it would make a significant impact on the understanding of the number system by primary children. Alison Lurie (2014), in her obituary of Dienes, includes a quote by Dienes that demonstrates his expectation of its success:

"How could a child learn what the Base 10 is, if he is not familiar with other number systems."

The theory was that the children would gain experience of working in a variety of different bases through the use of the blocks. The pupils would then use Base 10 blocks and this would result in a conceptual understanding of the number system. The theory was so compelling that both myself and many of my peers at college made use of this resource on our teaching practices and were confident that it would prove to be successful. Regrettably, instead of this outcome, as time went on it became increasingly clear that working in different bases had resulted in creating great confusion among the learners. This was a big disappointment but after I qualified and taught maths I continued to make use of Dienes MAB in the diminishing hope that an increased understanding of the number system would eventually come to fruition. This hope failed to materialise and so I then removed the Dienes material from the classroom in all of the bases except Base 10. Over the following months I discussed this with other primary teachers and discovered that the vast majority of them had also only retained Base 10 blocks.

"Researchers found that college students who learned a mathematical concept with concrete examples couldn't apply that knowledge to new situations."

Cuisenaire Rods

I first encountered Cuisenaire at teacher training college. Whilst the lecturers strongly recommended its use, I felt sceptical about it from the outset. There was a mixed reception from my peers. There was a considerable amount of support for it although a minority were unconvinced that it would prove to be an effective resource. I have continued to consistently have reservations about it and have not used it since qualifying as a teacher. One of the disadvantages of its use is that it entails a redundant raft of learning as the learners are required to memorise the value of all of the coloured rods. This information is of no use beyond activities directly related to the use of Cuisenaire.

One of the difficulties of its use was highlighted by Martin Hughes in *Children and Number* (1986). In a classroom observation Hughes observed a boy who rapidly added two numbers accurately with the use of formal methods and then got in a muddle as he tried to assemble his rods to give the impression that his answer had been obtained with the use of the rods in order to please his teacher.

In a situation in which the learners are effectively using concrete material, how does the teacher move them on to a conceptual understanding of the topic?

Consideration will now be given to the question contained in the introduction.

The question implies that there is a natural progression from the use of concrete materials to the acquisition of a conceptual understanding. The first point that needs to be established is whether this assumption is valid or not. Vladmir Sloutsky

(2008) points out that this theory has never been tested:

"The belief in using concrete examples is very deeply ingrained and hasn't been questioned or tested."

Kevin Delaney (2001) expresses a similar view:

"…the widespread belief…that practical or concrete experiences are essential for children to learn mathematical concepts. Such a belief permeated teacher education and teacher resources for decades…"

Research into this issue was undertaken by Kath Hart and her team in 1983-5 and one of their main findings involved:

"…the difficulties many children had in moving from the concrete or pictorial representations to the more formal (general) aspects of mathematics and the ability to link the stages of the teaching learning process."

Vladimir Slousky conducted research at Ohio State University and he concluded that:

"Researchers found that college students who learned a mathematical concept with concrete examples couldn't apply that knowledge to new situations."

Is it advisable to make widespread use of Cuisenaire Rods and Dienes MAB?

In trying to reach an objective assessment of the efficiency of the use of Cuisenaire Rods a difficulty arises in that it can neither be proved that it is effective or conversely that it is ineffective. Hence it is impossible to prove a case either way. If there is evidence of a considerable improvement in a child's mathematical attainment, a causal link would need to be established between the improvement and the use of Cuisenaire as it is possible that the improvement would be due to factors independent of its use.

In the case of Dienes MAB the discontinuation of its use by many experienced teachers suggests that it has been ineffective in a large number of classrooms. Furthermore, the confusion generated by the requirement of children to work in different bases suggest that its use would be counterproductive. It would perhaps be advisable to only retain the Base 10 materials.

Mark Pepper

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Making a Change: How the Entry Level Certificate gives students the opportunity for success

Kirsty Behan, **who spoke very passionately about the difference this has made in her school, outlines the hope that has come to her pupils from the inclusion of this qualification in KS 4.**

With the introduction of the new 'more challenging' GCSE, and with traditionally higher tier topics such as trigonometry and factorising quadratics sneaking down to foundation, came the worry for those students at the bottom end. In May 2017, it became apparent there were several students who would not be able to attain the lowest grade of a 1, these students were entered for the Entry Level Certificate in a bid to get them a mathematics qualification. This was a great start in beginning to provide those students with the provision and mathematics education they deserved.

All students should feel success in the mathematics classroom, regardless of their abilities, difficulties or backgrounds.

All students should feel success in the mathematics classroom, regardless of their abilities, difficulties or backgrounds. Yet, I have felt for the past few years for those in the lowest attainment classes, that students feel like failures. Partly, this is down to the change in curriculum, partly down to the pressure schools face to produce 'passing' grades, Cs/4s and above, and partly down to not having a way to allow them to feel this success. This year, I introduced the Entry Level Certificate into the Key Stage 4 curriculum at our school for those that are the lowest attaining. Whilst, traditionally this was used for only those who were struggling to achieve at grade 1. I made a change and, with the support of the head of mathematics and the SLT link, all of the students in the lower attaining sets were given the opportunity to take the Entry Level Certificate during year 11 and year 10. Next year, only students in year 10, and year 11 students who did not pass Entry Level 3 when they were in year 10, will sit the certificate. Now, to some this may seem like a pointless exercise that takes time away from the vital time they could be spending covering the vast amount of content that is now in the GCSE, particularly for those students who are more likely to achieve a grade 1 and beyond. However, I would argue that it is not pointless nor does it take away a lot of time from the curriculum.

Why is it good for the students?

Now, it may just be me, but I feel that many of my students who are placed in 'bottom' sets go through secondary school and rarely experience success. Every assessment, for some across many subjects, they 'fail' and continually achieve low marks. When they reach Key Stage 4, these students who have found maths challenging will now be faced with mocks that they cannot achieve in and low predicted grades. For me, the Entry Level Certificate was a chance to allow students to experience success and gain a qualification. Additionally, each student has three opportunities to take each test and task at each level, giving them time to improve and focus on improving their mathematical skills with less pressure. I timetabled Entry Level exams with the mock exams, allowing students to focus on passing the Entry Level just after they had done their mocks. This allowed students to feel success after sitting a more challenging GCSE mock and gave them a confidence boost after, a potential, confidence setback.

these students who have found maths challenging will now be faced with mocks that they cannot achieve in and low predicted grades

Why is it good for the teacher?

As our school has chosen Edexcel as our exam board it made sense to choose the Edexcel Entry Level Certificate. In 2017 the Entry Level Certificate has been redesigned to support students build toward the GCSE. Each of the levels is comprised of a non-calculator test and a task, with Entry Level 3 having an additional calculator test. These tests and tasks can be sat at any time during the examination window, which is normally from January to May, giving the teacher and students plenty of flexibility. Tests and tasks can be taken in the classroom with no time restrictions, meaning the students and teachers can easily fit these into their curriculum. The skills that are tested in the Entry Level exams are the basis for the GCSE, skills such as rounding, calculating, basic geometry and data skills, so do not need to be taught as separate 'entry level skills.' The only additional preparation I did with the students was practicing and learning how to approach the tasks. These are slightly different from anything I would have taught students, so it was necessary for their success that we spent time learning how to organise their thoughts, answer unseen problems and explain their reasoning; which ultimately are needed for the GCSE anyway. Lastly, thoughts on marking and administration. The Entry Level tests and tasks are short, with a maximum of 20 marks being awarded in a single paper so they are quick to mark and moderate, if more than one teacher is delivering the course. All the exam board require are the submission of the marks at the highest level the student achieved in that exam period and a sample of your marking to check.

One student in my class this year has said that the Entry Level has given her 'hope' and that she is proud of what she has achieved in maths.

One student in my class this year has said that the Entry Level has given her 'hope' and that she is proud of what she has achieved in maths. She will leave year 10 with hope for year 11 and the motivation to continue to succeed. For a small change in my practice, at minimal cost to the school, and a few hours of my time I think that is worth it.

Is That a Big Number?

Alan Edmiston is greatly impressed by this work by Andrew Elliot, read his review to find out why this was such a lovely read.

A. Elliot, Oxford University Press

From time to time *Equals* is asked to review books that may be relevant to the field of mathematics teaching. I accepted the invitation to review 'Is That a Big Number?' By Andrew Elliot as the request came just before the Summer break when I was looking for something to read and am I so glad it came before we left for the Lake District. I really love this book and plan to share some of Andrew's insights with the many children I have the pleasure of teaching.

This book was more than a good read – it is a resource I will use for many years to come.

This book was more than a good read – it is a resource I will use for many years to come. The writing is accessible, thoughtful and, on an interesting note, very, very positive about the role of numbers to help make informed decisions in all areas of daily life. On page 241 Andrew states that he has written a 'book of practical numeracy' rather than one about numbers. To me this phrase sums up why it is so useful. There have been many times while teaching that I have done just that; helped my pupils to apply a sense of numeracy to the problems they may face in their daily lives.

he has written a, 'book of practical numeracy' rather than one about number

Another plus for me is that very early on he also quotes from Staneslave Dehene, another author who I respect, using the term 'Number Sense', to describe humanities ability to comprehend numbers in an intuitive way, one that is readily available to our thought processes. To help other develop such as sense his book is built around the following five tools:

- Landmark numbers: using memorable numbers to make comparisons,
- Visualisation: using imagination to picture numbers in helpful contexts,
- Divide and conquer: breaking numbers down into smaller parts to work with,
- Ratio: making numbers smaller by expressing them as a proportion of some helpful base,
- Logs: dealing with numbers by measuring proportionate variation and not absolute difference.

Using the above tools as a way of dealing with large values and difficult to compare numbers, Andrew Elliot fills pages after page with points that not only make you think but also send your thoughts off in strange and wonderful tangents.

The density of Saturn is one I have used time and time again and the way he calculates how much Mr Darcy would actually be worth today was wonderful. If you want to know, his annual income today would be £720, 000!

Each chapter also begins with an interesting series of quiz questions such as: Which of these is the fastest?

Top speed attained by a human-powered aircraft
Top speed of a giraffe
Top speed attained by a human-powered watercraft
Top speed of a Great White Shark

It's the second @ 52 km/h

Agreed consistent methods for teaching mathematics at the Russell Education Trust (RET)

Teresa Robinson (Lead Mathematics Advisor for the Russell Education Trust) shares some of the successful strategies her schools have been using to support the least able.

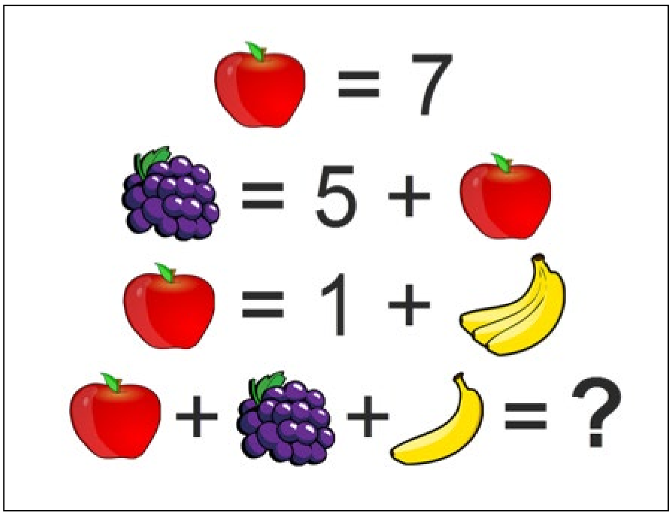
With the introduction of the new GCSE (9-1) the mathematics teachers at the Russell Education Trust (RET) recognised that the bar for mathematical attainment had been raised. Students would be expected to have a deeper understanding of mathematical concepts, be able to recognise and understand connections and links and successfully apply their knowledge to solve unfamiliar and multistep problems. These are skills that SEND and lower ability students often find particularly challenging. This, combined with the additional content, means that teaching time is even more precious and must be used effectively. All of the mathematics teachers in the Trust met to discuss and agree a set of consistent mathematical methods. We felt that if students experienced consistent methods throughout their school journey they would be more able to retain and build on their prior knowledge and be better placed to become confident mathematicians.

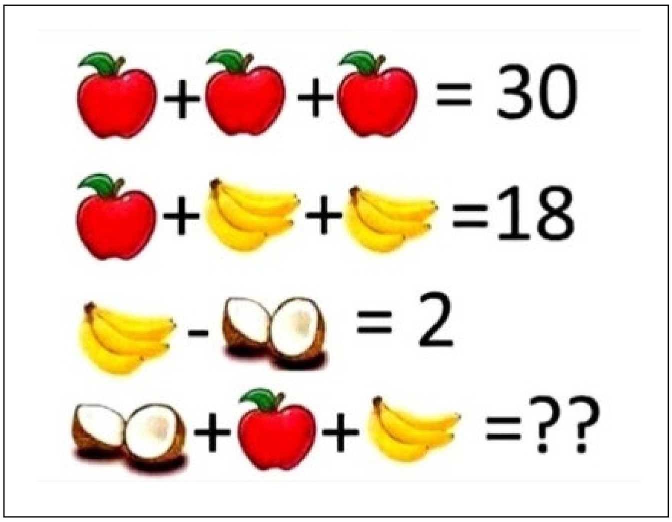
The task of agreeing consistent methods was both enlightening and powerful. Teachers willingly shared and justified their preferred methods, were receptive to alternative approaches and agreements were reached with surprising ease. It is a task that I would highly recommend to all mathematics departments.

Teaching resources to promote the agreed methods were created and shared by teachers in all of the schools in the Trust. Our consistent methods are not set in stone; if a student successfully applies a method previously learnt then they will be encouraged to continue using this approach, similarly, if a student fails to grasp our chosen methods then alternative approaches are considered.

The following is one example of our consistent methods. This is the agreed approach for using bars to develop a deep understanding of algebraic equations.

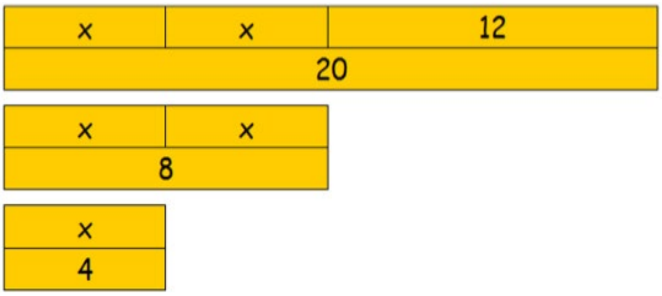
To introduce the topic of equations students are challenged to solve puzzles such as these:





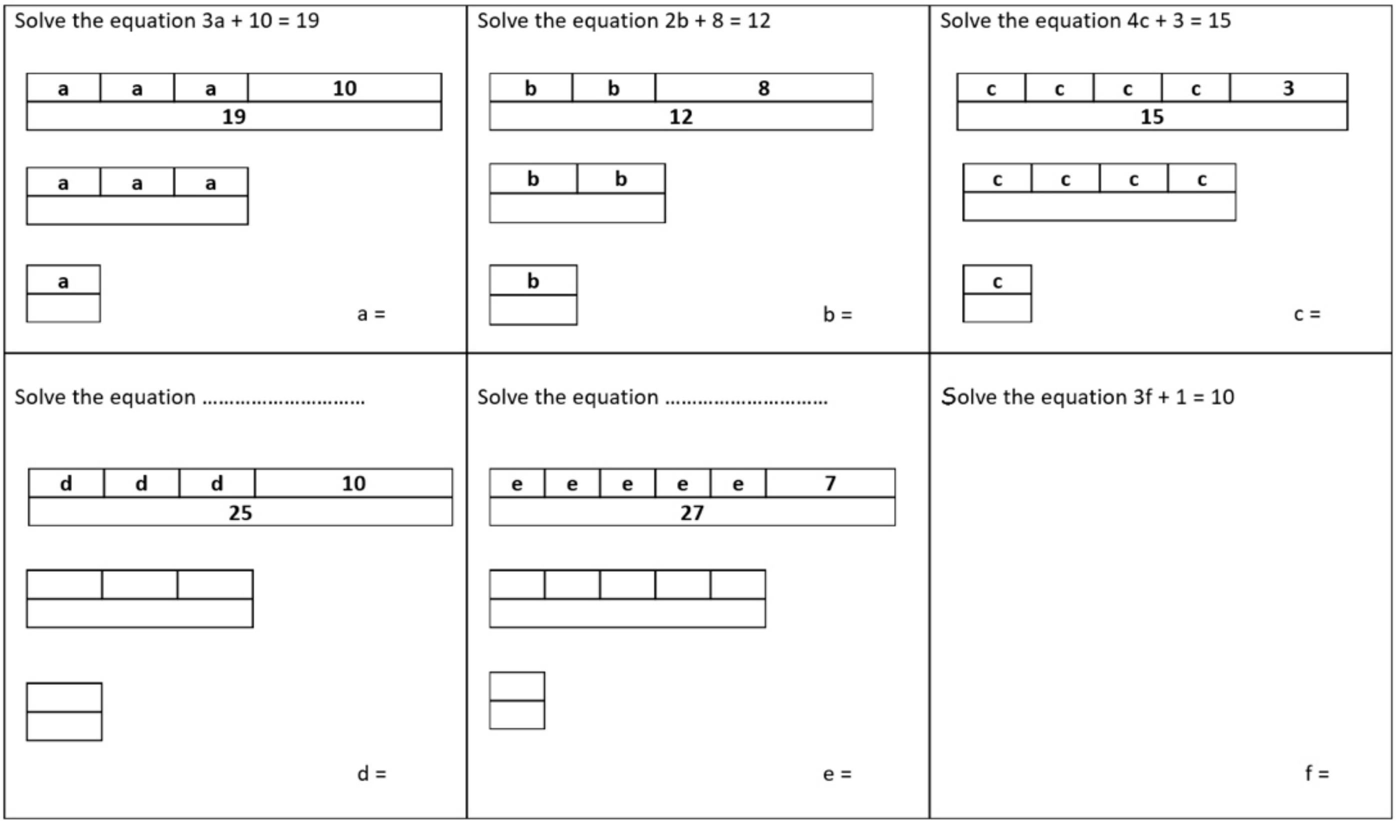
We recognise the advantages for SEND and less able students to develop their mathematical understanding through the use of concrete, pictorial and abstract approaches. To facilitate this, we use bars as an effective visual aid to introduce the balance method and to appreciate why inverses are so important.

Students may be asked why the top bar of this diagram represents the equation 2x + 12 = 20 and how this sequence of bars helps to solve the equation.

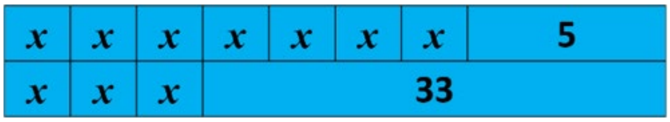


Students may use mini whiteboards to write down the equation represented by different bars, such as the one below:

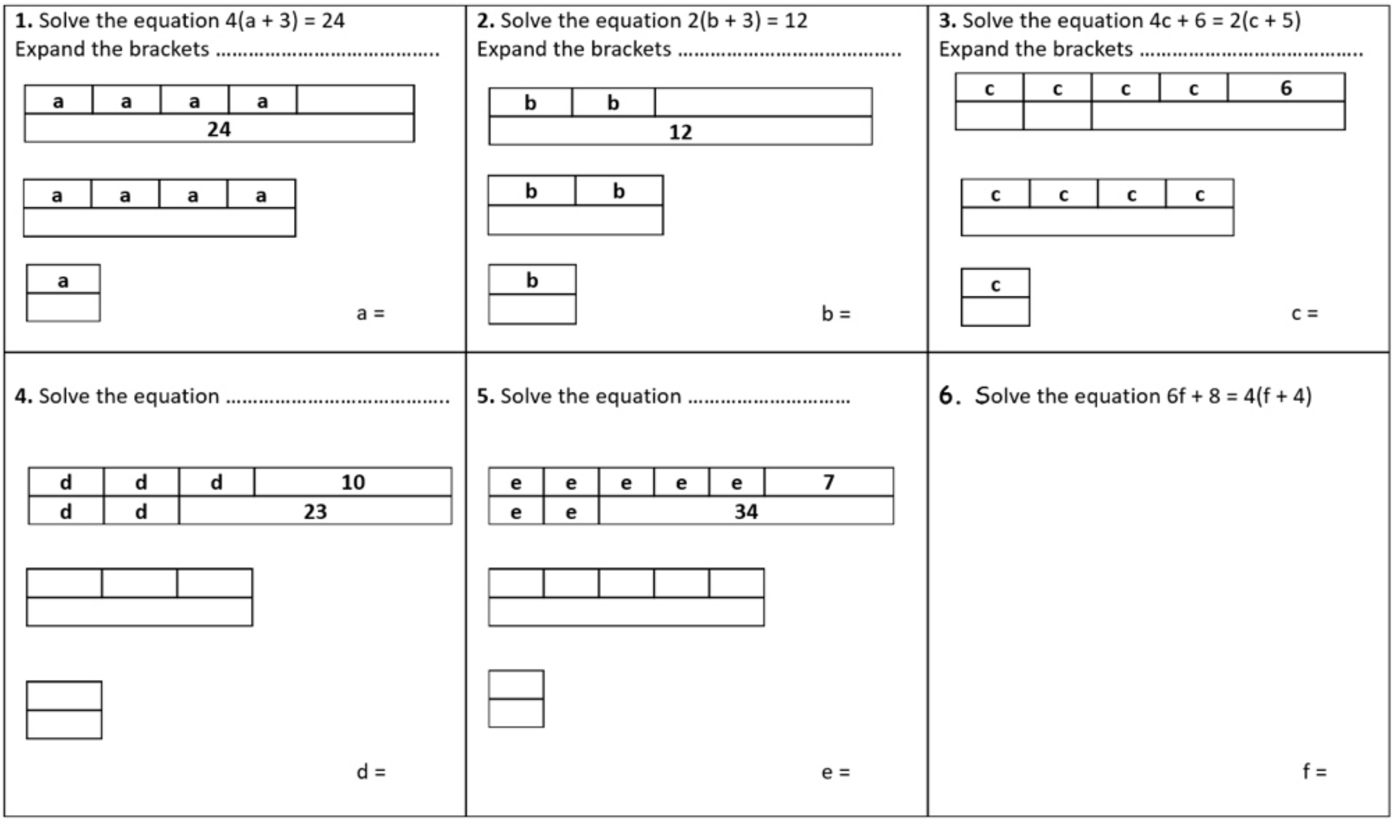
The following scaffolded worksheet was created to develop a deep understanding using bars before introducing a formal method of solving equations.



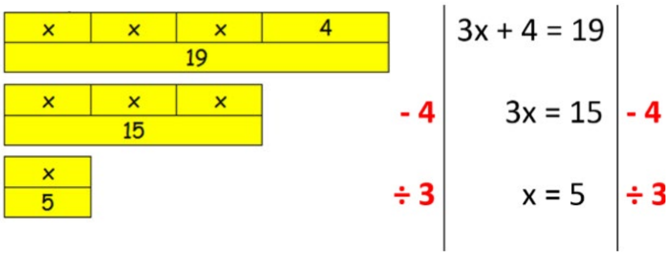
Students are also to be shown how bars can be used to solve an equation with an unknown on both sides. For example, students may be asked to explain why the bars below represent the equation 7x + 5 = 3x + 33 and how the sequence of bars can be used to solve the equation.



A similar worksheet to the one above was created to further develop understanding.

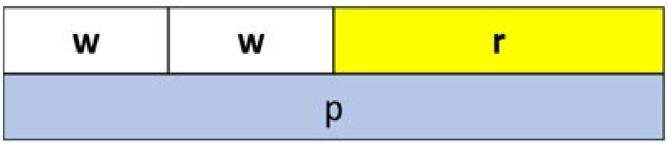


We found that the bars promote a good understanding of the balance method and the use of inverses when both methods are illustrated side by side, as shown below:



Once a more formal method of solving equations is established students can then be set equations involving negative values, this is a logical progression. The use of bars was promoted to support SEND and lower ability students and has been used successfully with students of all abilities.

Bar modelling is also promoted as a way to introduce the rearrangement of formulae. For example, students may be asked to write down all the different formulae that can be represented by the bars below.



Some of the other topics for which we have found consistent methods and collaborative planning to be particularly successful are: use of bars for teaching ratio and proportion, calculations with fractions and percentages, and use of grids for expanding and factorising quadratic functions. We have also agreed teaching methods for multiplying and dividing decimals and finding nth terms of linear and quadratic sequences.

SEND and lower ability students have benefitted from the consistent approach between one teacher and another, the focus on understanding and the high-quality resources provided. Teachers have welcomed the opportunity to discuss pedagogy, participate in collaborative planning and share resources. We shall continue to review and improve our consistent methods in order to ensure that students enjoy mathematics and achieve to the best of their ability.