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Adventures in shape and space – and time

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Introduction

As a youth entering the sixth form to study Mathematics, Further Mathematics and Physics I enjoyed the riches of the school's mathematics library and in particular three books which appealed to me, *A mathematician's apology* [1], *A book of curves* [2] and *On growth and form* [3].

Hardy's book [1] is one that an impressionable, young mathematician should not read unguided. It left me with the impression that the proper pursuit of mathematics was as a pure subject, of no use or application, to be studied for its own sake; to my regret, I held to this view for several years before finally being able to shake it off through teaching Newtonian mechanics. Looking across mathematics teaching today I seem to observe great interest in geometry, number and algebra 'curiosities' that are rooted entirely in mathematics. This in itself is no bad thing, since it clearly draws us and our students into the fascinating world of mathematics. But what of the applications of mathematics? Might they be equally fascinating? Surely we do not want to lure our students into Hardy's trap?

Lockwood's book [2] only reinforced my youthful impression, its beautiful and austere envelopes of curves presenting a black and white landscape of no apparent application but nevertheless inviting of exploration. So much did I admire this book and the way in which it combined Euclidean and coordinate geometry, my then two favourite areas of mathematics, that I bought my own copy with the fifth form English Literature Prize, much to the distaste of the Head of English. I still possess it and have used it frequently in my teaching to show students how beautiful mathematical curves may be created through the careful and accurate use of ruler, set square and compasses prior to teaching the traditional constructions. And yes, I know that any computer-based geometry package can do the same more quickly and probably with greater emphasis upon the mathematics through using the package but personally I prefer the hands-on approach to come first.

D'Arcy Thompson's book [3] was something of an enigma to me, a book applying mathematics to biology, a subject which I then thought, in my ignorance, completely without mathematics. Yet the diagrams held a fascination, the same fascination as Lockwood's book of curves, but these were diagrams of real tangible things, animals to be seen in everyday life. I did not read the book, the first page terrified me; quotes in German, Latin and Italian, were each used without translation. But the diagrams, oh the diagrams, they continued to draw me back over my sixth form career and still do to this day, repeatedly prompting me to ask about the mathematics in biology and, as a consequence, other subjects.

Common to these early experience of mathematics was geometry, Euclidean and coordinate. And the fascination with geometry has remained, particularly as shape and space, and especially as it appears in other subjects. So let us begin our adventure in shape and space – and time – looking at some of the mathematics in biology and, on the way, in other subjects as well.

Part 1: Of mice, elephants and other animals

A well-known (?) theorem:

Small mammals are furry or eat a great deal or both.

Proof:

Every picture you see of a small mature mammal, such as a mouse, shows it to be furry. And mice eat a quarter of their body weight every day. “That isn’t much”, I hear you say. But wait, I weigh about 80 kg. The same proportion of my weight is 20 kg. If I ate that much food every day I would be clinically obese or, and more likely, dead.

The question is, why? Consider a multilink cube as the model of a mouse. Its dimensions are 1 unit by 1 unit by 1 unit. Hence the surface area is 6 square units and the volume 1 cubic unit. Thus the surface area to volume ratio is 6:1. The mouse is big on the outside compared to the inside. In consequence though it can take in very little food at any one time it is losing heat quite rapidly all the time. Hence it must either be insulated, i.e. be furry, or it must eat a great deal.

Consider now a cube of side 6 units. Its surface area is 216 square units and its volume is 216 cubic units, giving a surface area to volume ratio of 1:1. Therefore as we scale up the dimensions of our cube, the surface area to volume ratio decreases. Our animal becomes bigger on the inside than it is on the outside. As it eats, it is generating heat within itself which it must somehow disperse. Hence we have a corollary to our theorem:

Elephants are walking radiators.

Pictures of elephants quickly confirm this, large ears and an excessively wrinkled skin that act as dispersers of heat in much the same way as the metal flanges of a radiator.

If, as biologists do, we take a typical dimension of our organism to be l, then the surface area (SA) is proportional to l^2 and the volume (V) is proportional to l^3, giving us:

$$\frac{SA}{V} \propto \frac{1}{l}. \qquad (1)$$

This relationship is not only in GCSE Biology, but has further consequences in addition to those we have just explored.

An immediate consequence is an extension of one that we have seen already: we cannot simply scale up. Thus if we scale up from a mouse to a human being, the digestive system has to evolve in some way so as to enable us to eat proportionately less but more efficiently and this means that the surface area of the gut has to be increased. Evolution has done this by providing the intestines with lots of little projections upon which are further projections called villi. A lovely article in [4] showed how 14-15 year-old mathematics students had explored the concept of surface area to volume ratio by exploring the potential shapes for villi. Similarly, the average capacity of the human lungs is about 6 litres but the surface area of the lungs, if they were to be 'unfolded', is between 50 m^2 and 100 m^2 because of the need to make the transfer of oxygen from the inhaled air into the blood stream as efficient as possible. To be large an organism has to be more complicated; it has to have adapted more via evolution than its smaller counterparts.

In an article, 'On being the right size', the biologist J.B.S. Haldane [5] set out some further intriguing outcomes amongst which is one that we may not wish to experiment about but which does challenge our intuition:

> "You can drop a mouse down a thousand yard mine shaft; and, on arriving at the bottom it gets a slight shock and walks away, provided the ground is fairly soft. A rat is killed, a man is broken, a horse splashes."

We tend to think that all objects falling under gravity will behave in the same way and this line of thought is encouraged because we frequently neglect air resistance, but in such an experiment as Haldane describes it would be foolish to do so. Air resistance varies as the surface area, i.e. as the square of an appropriate dimension; the driving force, the weight, varies as the volume, i.e. as the cube of the dimension. The body of a mouse is about 7 cm. long, that of man about 180 cm.. Therefore the mouse is over 20 times smaller than a man and the resisting force is proportionately over 20 times larger than the driving force. So terminal velocity is reached sooner and is far less. In essence our mouse 'floats' down at a constant but low speed for a very large part of the descent, whereas our man plummets down at an increasingly high rate.

Part 2: The inverse square law

Several years ago, a colleague, Dr. Tony Orton, and I were researching an article entitled, 'Science and Mathematics: a relationship in need of counselling?' [6]. One of the outcomes of the research was that, at that time, there was little actual mathematics in the A-level Physics questions as opposed to what was listed in the syllabus, whereas the A-levels Biology and Geography demanded arguably more use of mathematics in the form of statistics to be used in their respective field work exercises and had very mathematically demanding syllabi. But for me, one small, curious fact emerged. Whilst there was no science in the Key Stage (KS) mathematics tests (and how could there be, the tests were about mathematics not science) in the higher level KS3 mathematics paper, Newton's Law of Gravitation appeared quite regularly.

The questions took the form of presenting the inverse square law:

$$F = G\frac{m_1 m_2}{d^2}, \tag{2}$$

typically giving the masses of the Sun and the Earth, their distance apart, the value of the gravitational constant, all in standard form, and asking the candidate to calculate the force between the two bodies, giving the answer in standard form. This is an example of calculation for calculation's sake, something which in my view makes mathematics unpopular and open to the challenge, "Why am I doing this?"

However, other subjects do expect that their students will know the inverse square law and, more importantly, be able to calculate and interpret:

> Students should understand and use inverse proportion – the inverse square law and light intensity in the context of photosynthesis. (AQA GCSE Biology). [7]

So students should understand that if the distance of a plant from a light source is doubled the efficiency of the photosynthesis is reduced to a quarter of its original value. This is calculation with purpose and meaning.

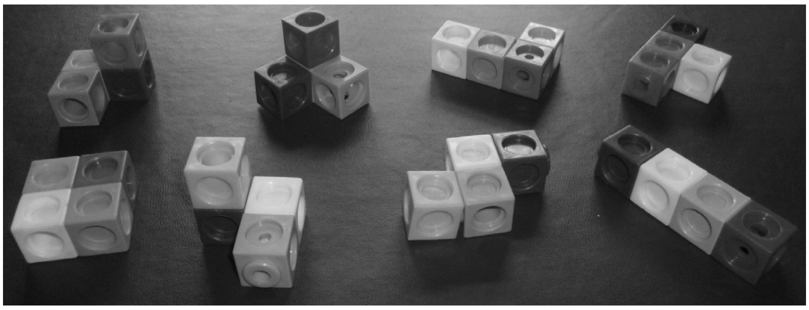
But there are also examples of the inverse square law in geography. If we replace the masses in (2) above with the populations of two towns or cities that are a distance d apart, and take a different constant of proportionality then we have:

$$F = K\frac{p_1 p_2}{d^2}, \tag{3}$$

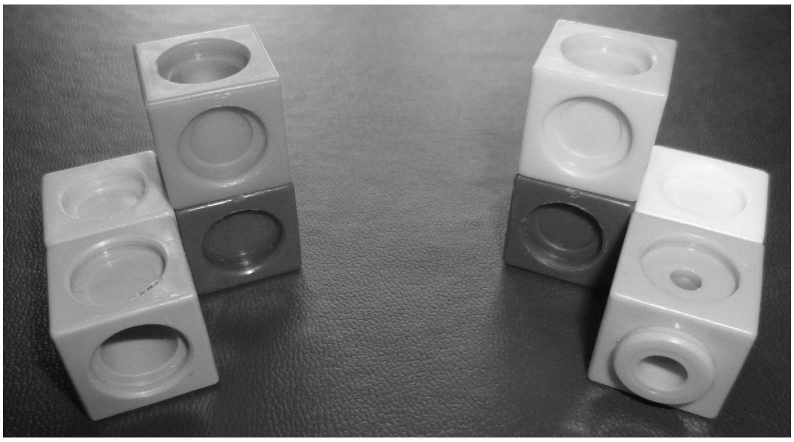
which may be taken to represent a measure of the 'attraction' or the 'interaction' between the two towns or cities. In his excellent book, *Pattern and Place* [8], Keith Selkirk used this model, and one with d replacing d^2, to investigate the sequence of building the motorways in England. Whilst the models are found not to be particularly good ones, one thing that does stand out is that the M62, linking the conurbations of Manchester and West Yorkshire, should have been built much earlier than it was. For those of us who live in the North of England, this evokes memories of the recent government decisions concerning the so-called 'Northern Powerhouse'. Once again the North comes a poor second in terms of transport and economic development. And yes this is politics, but surely one of the things that we would want our students to be able to do is to use their mathematics in other subject areas and be able make judgements about such issues as political decision making.

Part 3: Playing with bricks

The Presidential Address is given at the annual conference of The Mathematical Association and this year, 2018, the annual conference was a part of the four-yearly British Congress of Mathematics Educators (BCME). As a part of the Address I asked the audience at BCME carry out a task using Multilink cubes, which make an excellent medium through which to bring some ideas about shape and space into the classroom. The task given was to make as many different solids as possible using four of the multilink cubes. I have used this task with Years 8 and 9 working in groups, as well as with school teachers, and the outcomes have been exactly as at BCME. The audience were seated at tables and most groups at the tables produced the eight solids shown in Photograph 1 below but had doubts about two of them being different. Those groups that produced seven had another one in reserve so to speak because they felt it was the same as one of the seven but were not quite sure. The 'debatable' pair are shown in Photograph 2.



PHOTOGRAPH 1



PHOTOGRAPH 2

The solids shown in Photograph 2 are mirror images of each other; or, to put it another way, they are left-hand and right-hand versions of each other. The question which troubles most groups is, are they the same or not? And mathematics is about being able to say what is the same and what is different and why. I would say that they are different because you cannot place one of the shapes exactly into the space occupied by the other shape. In chemistry it is sometimes possible for left-handed and right-handed molecules of the same compound to occur and this can cause unexpected and possibly tragic effects. The thalidomide tragedy was caused by left-hand and right-hand molecules behaving differently.

Another interesting point that is worth drawing students' attention to is that whilst all the shapes have the same volume, they do not all have the same surface area and what the difference is in construction between those with the greatest surface area and those with the least.

Following on from this task, the groups were asked to make penguins, four multilink cubes in a vertical line and, having made several penguins, predict their behaviour in the depths of the Antarctic winter. All the groups huddled their penguins together and then immediately saw that if those penguins on the outside of the huddle were to survive, then they must move in towards the middle of the huddle and those on the inside make the opposite journey.

Film of penguins huddling together in Antarctica, [9], shows precisely this behaviour, commenting that the temperature at the inside of the huddle can be 80° above the air temperature outside, which is 40° below zero. Further, the temperature inside the huddle can be such that the penguins need to cool off and the huddle breaks apart spontaneously into individual penguins.

So playing with bricks can reveal some important mathematical facts and equally important illustrations of applications in chemistry and biology of the structures that are made from the bricks.

Part 4: Walking back to the future

William Froude (1810-1879) was an engineer and naval architect. He took the first steps in the scientific design of hulls of ships. Essentially the problem is to determine at what speed a real hull of similar shape to a model hull will behave in the same way as the model. To assist him he introduced what we call today the Froude number, Fr. The Froude number is defined as the ratio of two forces, the inertial force, that given by $F = ma$, and the gravitational force, mg. Therefore we have:

$$Fr = \frac{ma}{mg} = \frac{a}{g}.$$

Using dimensional analysis we arrive at:

$$Fr = \frac{u^2}{lg},$$

where u is the speed of the ship and l is its length. Model and ship will behave the same at the same Froude number. Thus if the model is scaled up by a factor of four to create the ship, the ship will exhibit the same behaviour as the model when travelling at twice the speed. Their motions through the water are kinematically similar.

The idea of kinematically similar motions gives us another way of looking at the Froude number, a way that is based upon ideas about geometric similarity rather than forces. Two motions will be kinematically similar if one motion can be made identical to the other by changing the scale of the length or the scale of the time or both. For example, two simple pendulums of different lengths but with the same amplitude are kinematically similar. If the lengths of the pendulums are l_1 and l_2 then $l_2 = kl_1$. If the corresponding time intervals in the two pendulum systems are t_1 and t_2 then, by taking $t_2 = \sqrt{k}\,t_1$, we can make the motion of the second pendulum identical to the first.

So for two particles, P_1 and P_2, with position vectors, $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, moving with kinematically similar motions, following Duncan [10, p. 20], we have:

$$\mathbf{r}_2 = \lambda\mathbf{r}_1 \qquad \text{and} \qquad t_2 = \tau t_1.$$

In times δt_1 and δt_2, the position vectors change by $\delta\mathbf{r}_1$ and $\delta\mathbf{r}_2$, respectively, where, $\delta\mathbf{r}_2 = \lambda\delta\mathbf{r}_1$ and $\delta t_2 = \tau\delta t_1$. Hence:

$$\frac{\delta\mathbf{r}_2}{\delta t_2} = \frac{\lambda\delta\mathbf{r}_1}{\tau\delta t_1}.$$

And taking limits, we have:

$$\frac{d\mathbf{r}_2}{dt_2} = \frac{\lambda}{\tau}\cdot\frac{d\mathbf{r}_1}{dt_1}.$$

Similarly:

$$\frac{d^2\mathbf{r}_2}{dt_2^2} = \frac{\lambda}{\tau^2}\cdot\frac{d^2\mathbf{r}_1}{dt_1^2}.$$

These equations are true for the individual coordinates of P_1 and P_2, for example:

$$x_2 = \lambda x_1,\ t_2 = \tau t_1,\ \frac{dx_2}{dt_2} = \frac{\lambda}{\tau} \text{ and } \frac{d^2x_2}{dt_2^2} = \frac{\lambda}{\tau^2}\cdot\frac{d^2x_1}{dt_1^2}.$$

Therefore if l_1 and l_2 are corresponding linear dimensions of the paths of the two particles, u_1 and u_2 corresponding velocity components and a_1 and a_2 corresponding accelerations, then we have:

$$l_2 = \lambda l_1,\ u_2 = \frac{\lambda}{\tau}\cdot u_1 \text{ and } a_2 = \frac{\lambda}{\tau^2}\cdot a_1.$$

Combining these equations to eliminate the constants of proportionality we have:

$$\frac{u_2^2}{a_2l_2} = \frac{u_1^2}{a_1l_1}.$$

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$$\frac{u_2^2}{a_2 l_2} = \frac{u_1^2}{a_1 l_1}.$$

For motion across the surface of the Earth for animals and humans, the governing force is the force of gravity and hence the relevant acceleration is g and we have:

$$Fr = \frac{u^2}{gl}.$$

Therefore animals that are moving with the same Froude number will be moving in a kinematically similar fashion. That is they will be moving with the same gait. Alexander [11], working with data from extant animals, used this idea to estimate the speed of bipedal and quadrupedal dinosaurs from sets of footprints preserved in mudstones, the footprints giving the stride length. The estimates produced suggested that these particular dinosaurs walked rather slowly, bipedal at around 2.2 ms^{-1} and quadrupedal at about 1 ms^{-1}.

So we see that a piece of mathematics, fundamentally based upon geometry and ideas of similarity, projects us back into the past. Could some similar mathematics project us forward into the future? Consider the question, 'How fast can you walk?'.

There are several ways in which we might answer the question. We could initiate some trials where people walk over a measured distance as quickly as they can, timing them as they do so. These trials could be used to calculate an average walking speed. The trials could also help us see if there are differences between men and women in terms of average speed. Another method might be to look up the average speed of race walkers in various walking races. But these methods simply give us numbers. What we really want is a relationship connecting some of the essential variables involved in walking that will tell us about walking, that will be justified by the world of walking that we see around us and will enable some reliable predictions about walking to be made. This is what mathematical modelling is about and it is something that we have already seen in what has gone before, the model mouse, the model penguins, the relationship (1) between surface area and volume predicting the effects of falling freely under gravity until hitting the bottom!

So where to look for our model? A strange but productive place to look is the John Cleese sketch from the Monty Python series, The Ministry of Silly Walks. Type the name of the sketch into your search engine and YouTube will produce the clip! Periodically in his walk, Cleese balances on one leg and extends the other leg out in front of himself before planting his foot on the ground and rotating his body over that stationary foot. In effect his centre of gravity has rotated in a circular arc, with the centre of the circle being the planted foot and the radius being equal to length of his leg. I very much doubt that the originator of this model, Alexander [12], used John

Cleese as his inspiration, but the video clip provides an illustration of part of the model – and much humour!

So we will model walking as a series of circular arcs, with radius ℓ equal to the leg length, forward speed v, and the body being a particle positioned at the centre of gravity of the body, which is centred approximately between the hips. The two forces acting are the weight mg acting vertically downwards and the reaction R of the ground on the foot, vertically upwards (see Figure 1). Applying $F = ma$ radially inwards and noting that the acceleration towards the centre of the circle is $\frac{v^2}{\ell}$ we have:

$$mg - R = m\frac{v^2}{\ell}. \qquad (4)$$

When we try to walk too quickly, our gait changes and we beginning to run, and when we run there are periods of time when we do not have contact with the floor. Thus in order to be walking we must have contact with the floor, i.e. we must have $R > 0$. Rearranging (4) to give R and setting it to be greater than zero, we have:

$$R = mg - m\frac{v^2}{\ell} > 0.$$

And so we get:

$$g\ell > v^2.$$

That is we cannot walk any faster than $\sqrt{g\ell}$.

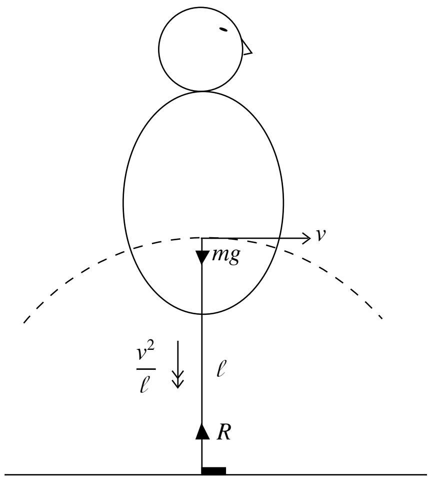

FIGURE 1

But is this in any way reasonable? We began with a method of walking that had remarkable similarities to a well-known comedy sketch! So let us try some numbers. My leg length is about 0.9 m and g is about 10 ms^{-2}, which means I cannot walk any faster than about 3 ms^{-1}. This is rather high, indeed race walkers do not walk this fast, but it is in the right area. Further, there is a dependency upon the leg length, which means that children cannot walk as fast as adults. This fact is demonstrated by the childhood experience of having to run to keep up with one's parents when they wanted to get somewhere quickly and upped their walking pace. But there is also a dependency upon the value of g the value of the acceleration due to gravity. Mankind has walked upon the surface of the Moon where the acceleration due to gravity is about one seventh what it is on Earth and videos of walking upon the Moon show that the astronauts did not walk but bounced along, the acceleration due to the Moon's gravity being too low to allow for walking as on the Earth's surface. If you view these videos, available on YouTube, you might also note that the astronauts lean forward at a fairly large angle and have difficulty in cornering. Does the low value of the acceleration due to gravity have anything to do with these phenomena?

We can take this a step further. Maybe not in my life time but certainly in the life time of our students, mankind will walk upon the surface of Mars. We know the acceleration due to gravity on Mars and therefore our formula, $\sqrt{g\ell}$, tells us how those astronauts will walk. Trivial that may be, but notice what we have done with our application of mathematics, we have now projected forward in time and in space, some adventure! So if you want to be a time-traveller, be an applied mathematician!

Part 5: Mary, Mary, quite contrary, how does your garden grow? With silver bells and cockle shells and Fibonacci all in a row!

Phyllotaxis is the study of the constructions made by the organs and parts of organs of plants, their origins and their functions in the environment. Whenever we look at plants we see patterns, phyllotactic patterns which have been constructed by the plant and its organs. Examples such as spirals are well known: spirals found in the seed heads of sunflowers, formed by the segments of the skins of pineapples, and formed on pine cones. The spirals are in two intertwined sets, clockwise and anticlockwise. The claim is made that the number of spirals clockwise and the number anticlockwise are successive Fibonacci numbers. But is this so? Other than photographs in various publications what evidence is there that this is the case? In the case of pine cones, there is quite a lot. According to Jean [13]:

- Using 4,290 cones from 10 species of pine found in California found only 74 (1.7%) deviated from the Fibonacci pattern [14];
- Of 12,750 observations on 650 species found in the literature over the last 150 years, in more than 92% of cases the Fibonacci sequence arises. Further in about 2% of cases the Lucas sequence was found [15].

Further evidence comes from Coxeter (1989) [16] who provides data on the arrangement of leaves on the twigs of trees in the form of fractions. The fraction is produced as follows. As we look along a twig, select a leaf, then count the number of leaves until we reach a leaf in exactly the same position as the selected leaf. This gives the denominator of each fraction. Now count the number of complete turns made about the twig between our chosen leaf and the target leaf. This gives numerator of each fraction. The fraction then represents the amount of turn around the twig between successive leaves. Therefore looking at the data below, there is half a turn along the twig of an elm between successive leaves, a third of turn between successive leaves on beech and hazel trees.

From [16]

Elm	$\frac{1}{2}$ turn	i.e. 180°
Beech, Hazel	$\frac{1}{3}$ turn	i.e. 120°
Oak, Apricot	$\frac{2}{5}$ turn	i.e. 144°
Poplar, Pear	$\frac{3}{8}$ turn	i.e. 127°
Willow, Almond	$\frac{5}{13}$ turn	i.e. 138.5°

The numerators and denominators are successive Fibonacci numbers and the fractions are known to approach a limit. Thus the angles should also approach a limit, the golden angle.

Consider a circle whose circumference is divided into two arcs, a and b, such that they are in the golden ratio, i.e.

$$\frac{a+b}{a} = \frac{a}{b}.$$

Rearranging and letting $x = \frac{a}{b}$, we obtain:

$$x^2 - x - 1 = 0.$$

Hence $$x \approx 1.618.$$

Thus the golden angle $$\psi = \frac{360°}{1.618}.$$

This apparent mathematical regularity occurring across so many different plants and other botanical structures suggests that there exists a mathematical model behind it all. The search for this model, allied to the biological processes promoting phyllotaxis, has been going on for over a century and there exist various proposals, see for instance [13] in which a whole book is devoted to the modelling of phyllotaxis. In 1868, Hofmeister [17] put forward a set of hypotheses which have been the building blocks of some of the models. The hypotheses are:

- The apex has an axis of symmetry;
- Primordia form at the edge of the apex and, due to the shoot's/ flower's growth, they move away radially from the centre with radial velocity only dependent on their distance to the edge of the apex;
- New primordia are formed periodically, the period is called the *plastochrone*;

- The incipient primordium forms in the largest available space left by the previous ones.

There are some terms here which require explanation. The apex is the tip of the growing shoot or branch, or the centre of the sunflower head. It is where the new growth is taking place. A primordium is the new organ that is forming at or near the apex, a leaf in the case of shoot or branch, a floret (which will become a seed) in the case of the sunflower head, a segment of a pine cone or a segment of the skin of a pineapple.

Figure 2 below was extracted from a screen dump in a presentation by Atela, Golé and Hotton [18]. It illustrates the head of a sunflower modelled as a series of concentric circles in the complex plane. The oldest primordium is z_3 and this will move out along the radius as the plant grows to become Z_4, similarly for z_2 and Z_3, z_1 and z_2, z_0 and Z_1. Seeking the largest space left by the previous primordia, as suggested by Hofmeister's fourth hypothesis, the newest primordium emerges as Z_0 at ✯. The angle that is turned through by the radius each time is called the angle of divergence.

Through a mapping repeatedly applied to the space in the image, Hotten [19] and Atela, Golé and Hotten [20] modelled the last of Hofmeister's hypotheses as the minimum of a repulsive potential energy. In other words, the previous primordia repel their newest neighbour so that it takes its place as far away from them as possible within the area where growth takes place, i.e. in the greatest space available.

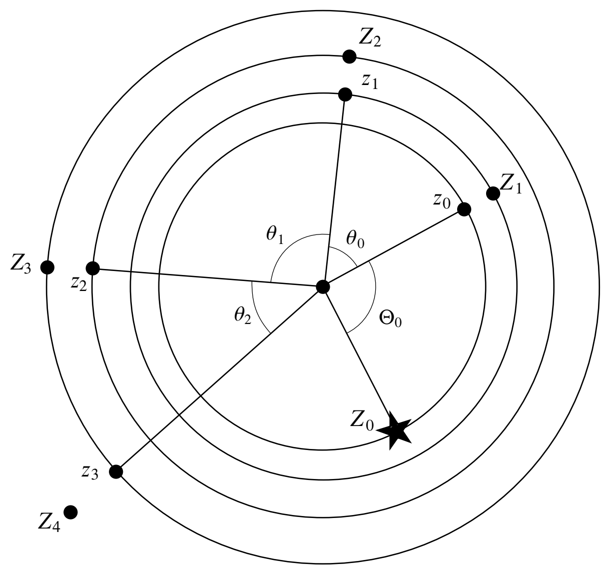

FIGURE 2

The concepts associated with mappings that we are interested in are fixed points and within fixed points, stable fixed points. Thus if we look at the mapping $x \to x^2$ defined on the real number line, then $x = 0$ and $x = 1$ are fixed points because under the mapping they do not change, physically they stay where they are. However, $x = 1$ is unstable because values of x close to $x = 1$ take values different from $x = 1$ when the mapping is repeatedly applied to them and move away from $x = 1$, no matter how close they are to $x = 1$. On the other hand, $x = 0$ is a stable fixed point because values of x near to $x = 0$ approach $x = 0$ when the mapping is repeated applied to them.

On repeatedly applying the mapping Hotton found that, for any given value of the plastochrone, the angle of divergence is a constant. Thus fixed points are the spirals that we see on the pine cones, pineapples and sunflowers, or the spiral of leaves around a twig. Further, there are two sets of spirals, one clockwise, one anti-clockwise, and the number of spirals in each set being successive Fibonacci numbers. The fixed points are stable and hence we see them repeatedly in nature.

As the plastochrone decreases, i.e. as primordia are generated faster and therefore nearer together in space as well as time, the mapping repeatedly bifurcates and at each bifurcation the number of spirals clockwise/anti-clockwise are successive Fibonacci numbers which increase with each bifurcation and the angle of divergence approaches the golden angle.

So we have a mathematical explanation of the occurrence of Fibonacci numbers and the golden ratio in the phyllotaxis that we see in pine cones, pineapples, sunflowers etc. But do we have a biological explanation? The literature suggests, and it is no more than a suggestion, that the new primordia, in seeking the largest space in which to grow and develop are in fact seeking out the greatest concentration of a growth hormone, called auxin, that is available to them in the apex or sunflower head. I personally find that there is an intuitive appeal in this suggestion, that physically the greatest concentration would be in the largest available space – but I am no biologist!

To support this model there is further evidence from two French physicists, Douady and Couder [21], who carried out an experiment essentially mimicking the hypotheses of Hofmeister. Magnetised droplets of oil are allowed to fall into the centre of a plane, horizontal, circular plate around the edge of which is set a circular magnet. When the droplets fall onto the plate, they are attracted to the rim of the plate by the magnet placed there. When the time interval between droplets is large, they simply move off in turn across the plate to opposite ends of the same diameter because they have little or no effect upon each other. This may be compared to the arrangement of leaves on a twig of an elm tree where there is half a turn between successive leaves. However as the time between droplets is decreased, then the droplets do have an effect upon each other because they are now nearer to each other and interconnected spirals, of successive Fibonacci numbers, may be seen to be formed on the plate as the droplets move off towards the rim. Video footage of their experiment may be found on [22].

Conclusion

And so concludes our adventure in shape and space – and time, but to what end have we made this adventure? I think it behoves us to remember that mathematics is not only the queen of science but the servant too. Newton did not do all his wonderful mathematics as an exercise in mathematics, it was driven by a purpose, to show how the heavens worked. We need to realise that there may be as much mathematics going on in the biology or the geography classroom next door as there is in ours, and sometimes may be much more and to greater purpose in terms of explaining how the real world works!

One of the claims made for the importance of mathematics and a reason for teaching it is its application to the real world. For politicians this would appear to amount to simple arithmetic but it is incredibly boring for our students (though essential I grant) and its application in the real world is rapidly diminishing as technology takes over and other skills become more important. It is, for example, more important that the person who deals with you in your bank has the personal skills to manage you and your business of the moment than to be able to do any of the calculations that your business requires. That is what the terminal mounted on the counter is for; the bank cannot afford mistakes arising from human error.

I would therefore argue that if we want to illustrate the importance of mathematics at a level appropriate to our pupils we should turn to the other subjects that they are studying, and be quite shameless about it. Yes, there is a cost. You have to be prepared to get to know something about the other subject, either by reading or, and better still, talking with colleagues who teach these other subjects. But this is an adventure in itself and one I have found well worth taking!

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