CENTRIFUGAL FORCE

Corkscrew—
Alton Towers Leisure Park

The Revolution — a 360° stomach churning looping coaster at Blackpool Pleasure Beach

Fact or Fiction?

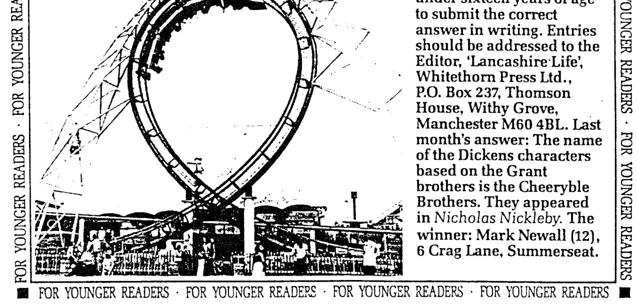
by M. D. Savage, University of Leeds

> **A woman saw her young son fighting.**
> **"Martin," she said, "I want you to stop this fighting; there is far too much friction in the world"**
> **Martin looked puzzled!**
> **"And you do know what friction is — don't you?" asked the woman.**
> **"Yes, Mum — Science Friction!"**

This story serves to illustrate how easy it is to misunderstand and, like young Martin, *get the wrong idea.* In Newtonian mechanics misconceptions and sources of confusion are commonplace — associated with, for example, the concepts of force, acceleration and Newton's laws of motion[1-10]. For the student of mechanics there is perhaps no other concept which causes quite so much confusion as that of *centrifugal force.* In this article we shall see how and why this confusion arises and what perhaps may be done to remedy the situation.

Now many people, including non scientists are familiar with the notion of centrifugal force. In fact, if the layman is asked "what do you experience when travelling at speed in a car around a corner?" — the answer almost invariably is centrifugal force! Recently younger readers of Lancashire Life were invited to compete for a £5 book token by answering a question relating to the 360° roller coaster at Blackpool's Pleasure Beach!

"What keeps the people, and the cars for that matter, from falling?" — the winning reply was — you've guessed it — centrifugal force!



FOR YOUNGER READERS · FOR YOUNGER READERS · FOR YO

Picture query: What keeps the people – and the cars for that matter – from falling when the 360 degree roller coaster at Blackpool's Pleasure Beach goes round the top?

A £5 book token will be sent to the first reader under sixteen years of age to submit the correct answer in writing. Entries should be addressed to the Editor, 'Lancashire Life', Whitethorn Press Ltd., P.O. Box 237, Thomson House, Withy Grove, Manchester M60 4BL. Last month's answer: The name of the Dickens characters based on the Grant brothers is the Cheeryble Brothers. They appeared in *Nicholas Nickleby.* The winner: Mark Newall (12), 6 Crag Lane, Summerseat.

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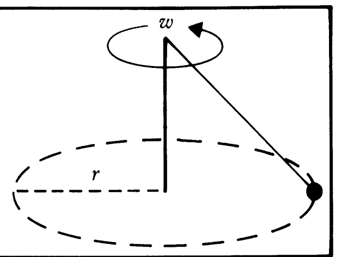
Fig. 1 *Courtesy of Lancashire Life*

Mechanics Questionnaire

At the University of LEEDS a mechanics questionnaire is given to first year science and engineering undergraduates to test their understanding of basic concepts. This questionnaire includes the following problems:

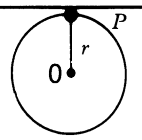
1.

A bob of mass m, is attached to a light and inextensible string and rotates in a horizontal circle of radius r with angular speed w about the vertical. Ignoring air resistance INSERT the forces acting on the bob.



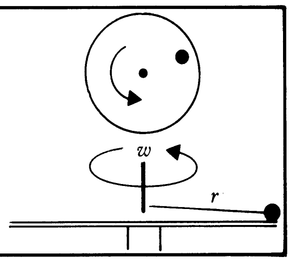
2.

A bob of mass m describes a vertical circle of radius r and centre O. If the string remains taut insert the forces acting when the bob is at its highest position P.



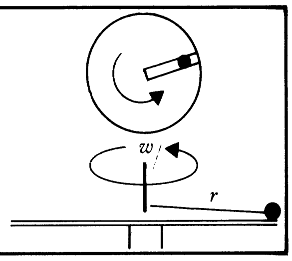
3.

A fly, of mass m, is at rest relative to a turntable which rotates with constant angular speed w. Insert the forces acting on the fly.



4.

A marble, of mass m, can move freely along a smooth groove in a turntable which rotates with constant angular speed w. Insert the forces acting on the marble.



Each year it is found that a number of students insist on including the so called centrifugal force, mrw^2, in addition to other forces such that their force diagram in question 1 would be as shown in Fig. 2. The inclusion of mrw^2 suggests a basic misconception about the nature of force.

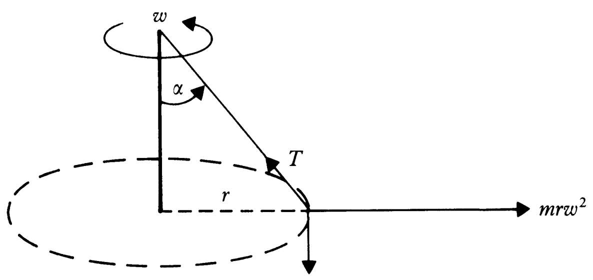


Fig. 2

Does the student know what force is and in particular does he or she understand what Newton meant by force? — i.e. what are the characteristic features of the **F** which appears in Newton's second Law?

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) \quad \text{or} \quad \mathbf{F} = m\mathbf{a} \qquad [1]$$

if mass m is constant and **a** is acceleration. Clearly force is that which causes acceleration and Newton's first law can be regarded as defining force in that way. Secondly, Newtonian forces include *contact forces* (i.e. reaction R, friction F, tension T) and *body forces* (i.e. gravity mg). Thirdly, force in the Newtonian sense *always involves an interaction between two bodies* and is the action that each exerts upon the other. Once this is realised then one source of confusion is immediately removed. It is now clear that mrw^2, though having the dimensions of force, is not a force — not in the Newtonian sense since there is no other body from which it arises. Hence if the student adopts the convention that force diagrams should include only Newtonian forces which thus constitute the **F** in equation [1] then he or she has a strategy for approaching problems such as 1–4 and proceeding towards a solution. For example in question 1, Figure 2, Newton's equations for motion in the vertical and radial directions are:

$$T\cos\alpha - mg = 0 \qquad [2]$$

$$-T\sin\alpha = m(\ddot{r} - rw^2) \qquad [3]$$

If r is constant then $\ddot{r} = 0$ and hence

$$T\sin\alpha = mrw^2 \qquad [3a]$$

A Paradox

Unfortunately this is not the end of the story — an additional source of confusion remains. Both the layman and the student of mechanics may still claim that centrifugal exists — why? — because he or she experiences it when cornering in a car or on a ride such as a chair-o-plane at Alton Towers. This is succinctly expressed in the following remarks by a sixth form student

Student to Maths teacher
"You have told us that centrifugal force does not exist! Why then do I experience an outward force (a *centrifugal force*) when taking a bend at speed in a car or on a chair-o-plane? Do I deny my experience?"

The source of confusion can often be traced to the fact that the mathematics teacher will state that centrifugal force does not exist (and therefore don't mention it again!!) whereas the physics teacher may well state that centrifugal force does exist and will include it on a force diagram — small wonder that the poor student is bewildered!

How is this paradox to be resolved?

How is the experience (of being thrown outwards) to be explained?

At this stage it is perhaps instructive to contrast the approach of the applied mathematician to circular motion with that of the physicist.

Applied Mathematician's Approach

Teachers of Applied Mathematics will, in general, adopt the approach mentioned earlier. Force diagrams will contain only Newtonian forces which emerge as a result of an observer in the rest frame (fixed in space) asking the question "which Newtonian forces are acting upon the body?" These forces then constitute the **F** in Newton's second law. For illustrative purposes we shall consider three problems mentioned earlier.

(a) For the conical pendulum, problem 1: Equations [2] and [3] provide a mathematical description of the motion as observed by such an observer, Figure 3.

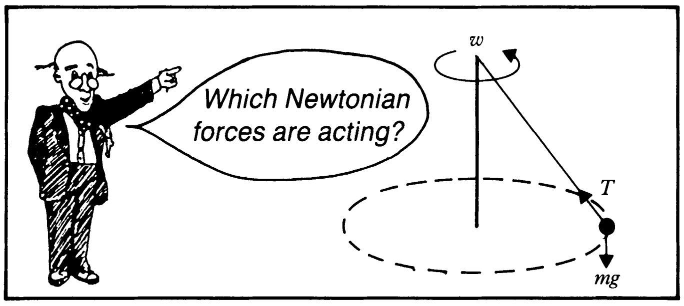


Fig. 3

(b) Fly on the turntable, problem 3; the Newtonian forces acting on the fly are weight mg, normal reaction R and friction F, acting radially inwards, Fig. 4.

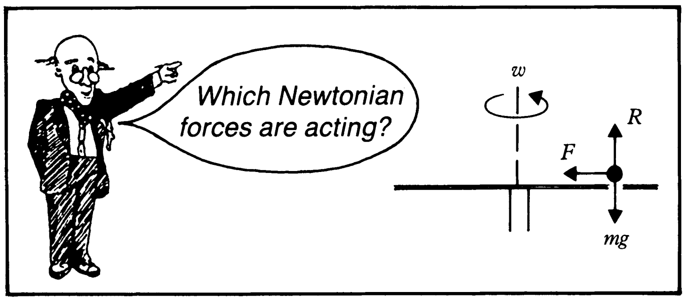


Fig. 4

The equations of motion in the vertical and radial directions are

$$R - mg = 0 \qquad [4]$$

$$-F = m(\ddot{r} - rw^2) \qquad [5]$$

where $\ddot{r} = 0$ if the fly is at rest relative to the turntable.

(c) Marble in the Groove, problem 4; there is no Newtonian force in the radial direction, Fig. 5.

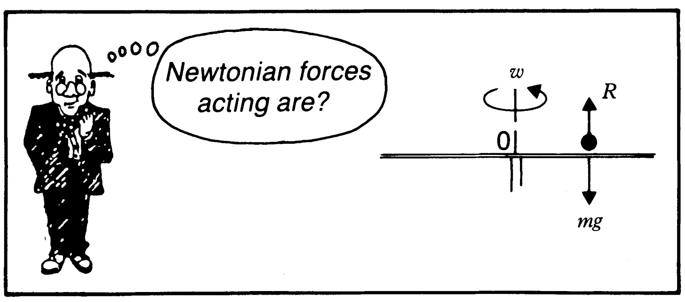


Fig. 5

Newton's equation then becomes

$$0 = m(\ddot{r} - rw^2) \qquad [6]$$

which can be solved to yield

$$r(t) = Ae^{wt} + Be^{-wt}$$

i.e. the radial distance of the marble from 0 increases exponentially.

Physicist's Approach

The physicist, like the applied mathematician, is fully aware of the preceding approach in which an observer in the rest frame asks "which Newtonian forces act on the body?" In addition, however, the physicist recognises that each of us has an experience of circular motion when cornering in a car or on a chair-o-plane — in fact whenever we are in the position of an observer in an accelerating frame of reference which rotates about the vertical with angular speed w see Figs 6 and 6a.

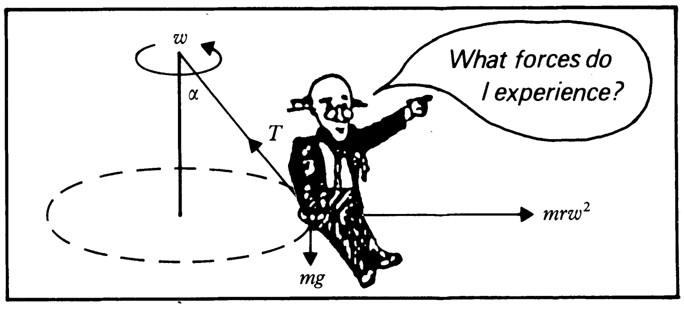


Fig. 6

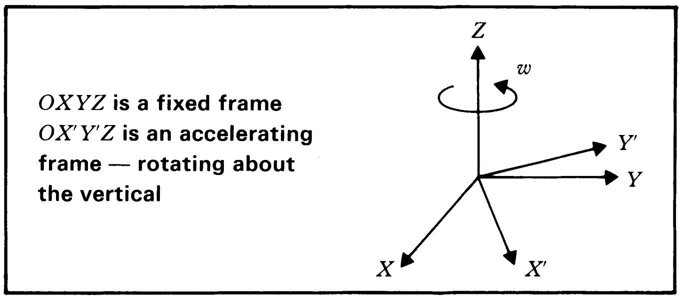


Fig. 6a

Such an observer experiences, or appears to experience a force-like effect acting radially outwards. This is commonly termed a centrifugal force, of magnitude mrw^2. Clearly it is not a Newtonian force (arising from another body) but it is referred to by some physicists as an inertial force* (ref. 7) — arising from the acceleration. The physicist seeks to explain the notion of centrifugal force and the experience of the observer in the rotating frame by considering motion relative to this rotating frame of reference. In particular, Newton's equation for motion in the radial direction, subject to a Newtonian force F_r and taking account of variations of r w.r.t time, is

$$F_r = m(\ddot{r} - rw^2) \qquad [7]$$

On rearranging, equation [7] becomes

$$m\ddot{r} = mrw^2 + F_r \qquad [8]$$

which describes motion (acceleration) relative to the rotating frame. Now we shall consider again each of the aforementioned examples.

(a) For the conical pendulum, Newton's equation for motion in the radial direction is

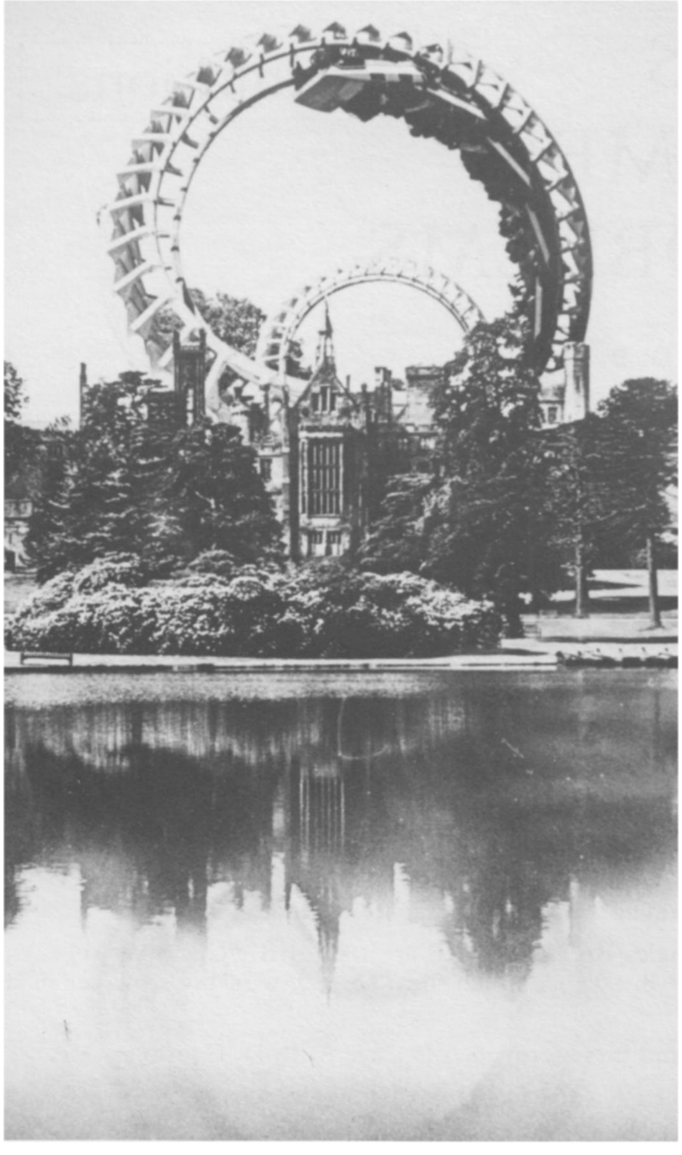
$$-T \sin \alpha = m(\ddot{r} - rw^2) \qquad [9]$$

which rearranges to yield

$$m\ddot{r} = mrw^2 - T \sin \alpha \qquad [10]$$

Equation [10] describes motion relative to an accelerating frame which rotates about the vertical with angular speed w, figure 6*a*. One interpretation of this equation is that the relative acceleration $\ddot{r}$ arises from the combined effect of both an inertial and a Newtonian force. For the case in which w is constant, then r is also constant and $\ddot{r} = 0$ such that mrw^2 balances $T \sin \alpha$ as illustrated in figure 6. A second interpretation is that

*Another example of inertial force is Coriolis force which is met with in geophysics and meteorology.



Towers/Corkscrew — Alton Towers Leisure Park

mrw^2 is a force-like effect which tends to accelerate the body radially outwards relative to the rotating frame.

(b) For the fly on the turntable, equation [5] can be rearranged to yield

$$m\ddot{r} = mrw^2 - F \qquad [11]$$

which indicates that the relative acceleration $\ddot{r}$ results from the competing effects of mrw^2 and friction. Clearly as w is increased the fly will remain at rest ($\ddot{r} = 0$) with F balancing mrw^2 until the limiting friction F_{max} is reached. The critical speed w_c is then given by

$$mrw_c^2 = F_{max} = \mu mg$$

$$\therefore w_c^2 = \frac{\mu g}{r} \qquad [12]$$

where μ is the coefficient of friction.

When $w > w_c$ then mrw^2 exceeds F_{max} and the fly starts to accelerate relative to the turntable. Once again mrw^2 can be interpreted as a force-like effect tending to accelerate the body radially outwards relative to the rotating frame.

(c) For the marble in the groove, problem 4, equation [6] rearranges to give

$$m\ddot{r} = mrw^2 \qquad [13]$$

Here it is seen that the effect of mrw^2 (the centrifugal effect!) is to cause the marble to accelerate outwards along the groove relative to the turntable.

These few examples illustrate that our everyday experience of force includes both Newtonian and inertial forces — and a clear distinction between them is essential. Though centrifugal force is not a Newtonian force it is not sufficient to assert that it does not therefore exist. In fact the experience of centrifugal force by an observer in a rotating frame is the experience of mrw^2 as a very real force-like effect which tends to cause an acceleration relative to the rotating frame. This is clearly seen by considering the equation for relative acceleration;

$$m\ddot{r} = mrw^2 + F_r$$

Conclusion

These examples have been used to construct two distinct approaches to circular motion. In one case the observer is in a fixed frame of reference considering Newtonian forces and actual accelerations, whereas in the other the observer is in a rotating frame of reference considering relative acceleration due to both Newtonian and inertial forces.

Both approaches are equally valid — yet in the author's opinion it is absurd to try to teach both simultaneously to the new student of mechanics — there is clearly far too much to digest with a very real danger that circular motion will remain a never-ending source of confusion. Surely it would be educationally more desirable to concentrate upon just one approach — at least initially until the student gains familiarity and confidence with the principles of Newtonian mechanics. In addition, the consideration of rotating frames of reference and motion relative to such frames only makes sense after the student has a clear understanding of Newton's equations in a fixed frame. It would therefore seem appropriate to make the following recommendation for the teaching of mechanics at the sixth form level.

Recommendation

A consistent approach to force and force diagrams should be adopted in both applied mathematics and physics at the sixth form level.

In particular,

(i) The question to be asked is "Which Newtonian forces are acting on the body?"

(ii) Force diagrams should then include only Newtonian forces which thus constitute the F in Newton's second law.

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Note

For those interested in investigations and practical work in mechanics, some of the materials described in this article form part of a mechanics kit — details of which can be obtained from the author.