Time for Some MATHEMATICS

Fig. 1 A group at work in Harlow

by Peter Ransom

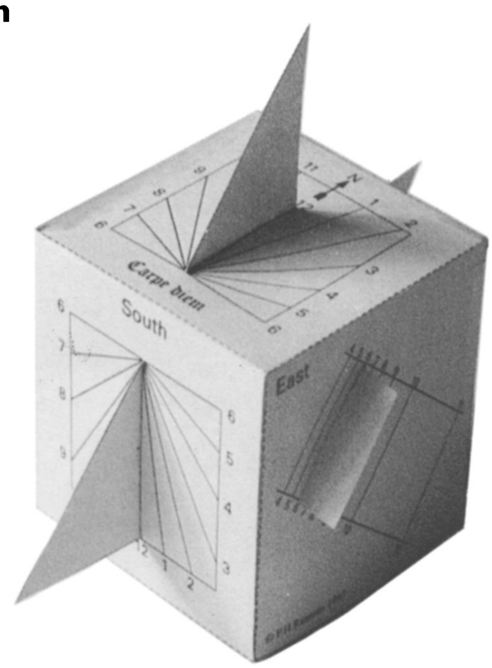

Fig. 2 Multiple Dial Block constructed from net

Introduction

I have an obsession with sundials, so when I noticed I was included in The Mathematical Association's Warwick Conference to 'do' a session on the history and mathematics of sundials it came as no surprise. For the past five years I have worked with year 9 pupils up to three times a year at Saturday morning Royal Institution Masterclasses which I call 'Fun with the Sun'. These I enjoyed doing at Newcastle upon Tyne, Harlow, Eastleigh, Portsmouth and Fareham. It is about time (sorry about the pun) something was set down on paper for others to read and use with pupils as those present at the conference workshop have experienced this directly. I have described some of the activities before (Ransom, 1993) but what appears here includes more practical work and some of it is suitable as a problem solving activity with small groups.

The mathematics covered involves

- nets (construction of a multiple sundial from a given net)
- language (parallel, net, latitude and longitude, inclined plane, rotation, angle, horizontal, estimate, circumference, etc.)
- arithmetic (four rules)
- estimation (of shadow time, and in reading graphs)
- reading graphs

Constructing the Multiple Dial

I think it important that young people who give up their time on a Saturday morning start with a practical activity. To break the ice and get them communicating I start my session by having them construct two multiple dials, one of which is given here. Of the two that they make, one is slightly easier and participants can decide on which to start.

The net for this dial is given on page 10. The dial is calibrated for a latitude of 51° North. If you intend using this dial at places where the latitude is more, then either cut along the appropriate line (indicated for 52°, 53° and 54° North) or ignore them totally and put the dial on an inclined plane of the appropriate angle ($x°$ for latitude $(51 + x)°$ North). More about why later.

Instructions for construction
Equipment needed:

Each pupil needs:
1 copy of the net photocopied onto card
1 pair of scissors

For every 5 pupils provide:
1 craft knife
1 ruler
sellotape or glue stick

1. Use a craft knife to slit the five thick lines on the dial faces.
   These are
   - the 12 line on the south face
   - the 12 line on the Carpe diem (Seize the day!) face
   - the 6 line on the west and east faces
   - the line pointing to G on the north face
2. Cut out the net, and cut along all the solid lines. This includes the line segment between the west face and A, the south face and B, pieces D and E.
3. Score all dashed lines on the lines and fold.
4. Score all dotted lines on the opposite side to the line, and fold.
5. Tuck
   - triangle A through Carpe diem slot
   - triangle B through south slot
   - rectangle C through west slot
   - rectangle D through east slot
   - triangle E through north slot
6. Tuck flaps F and G in to complete the multiple dial block.
   A bit of glue and/or tape makes it more robust.

A Bit of Theory

Following the practical work with a bit of theory brings everyone together, and allows the constructed model to be used to illustrate this. For a sundial to tell the time accurately it needs to be oriented correctly. This involves getting the gnomon (the bit which casts the shadow) parallel to the Earth's axis. The picture of the armillary sphere (from the Latin word for band) sundial helps explain why.

This sundial represents a 'see through' Earth. The arrow is the Earth's axis about which the sun appears to rotate. The brass plate with the hours on lies in the plane of the equator,

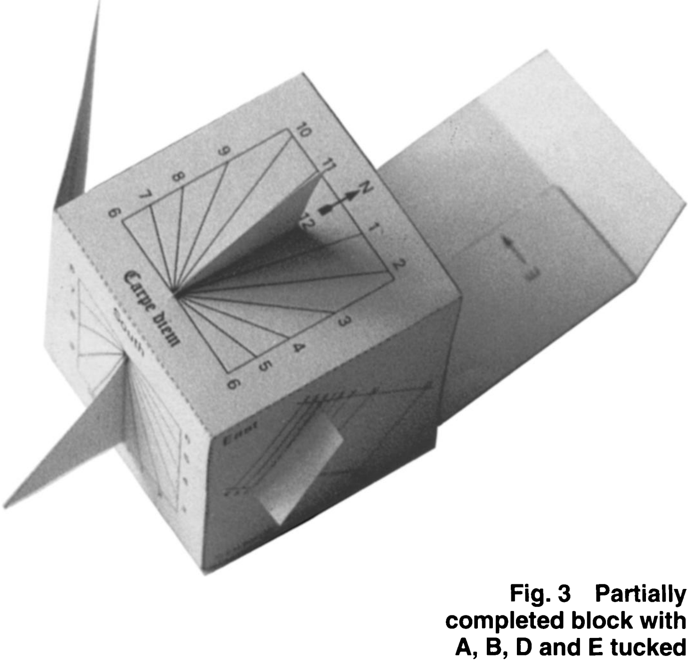

Fig. 3 Partially completed block with A, B, D and E tucked through their slots

Fig. 4 Armillary sphere sundial

and the hour divisions are equally spaced along this from 6 a.m. due west to 6 p.m. due east. When the sun rises in the east the shadow of the gnomon falls on the 6 a.m. line, and as the sun appears to rotate, the shadow moves along this plate reaching 6 p.m. when the sun is due west. We therefore realise that the sun appears to rotate through 360° in 24 hours, i.e. 15° in 1 hour, or 1° in 4 minutes.

Now this model is used for the basis of most sundials. It is worthwhile explaining that the gnomon of the sundial must be parallel to the Earth's axis, and illustrating this with the multiple sundial constructed.

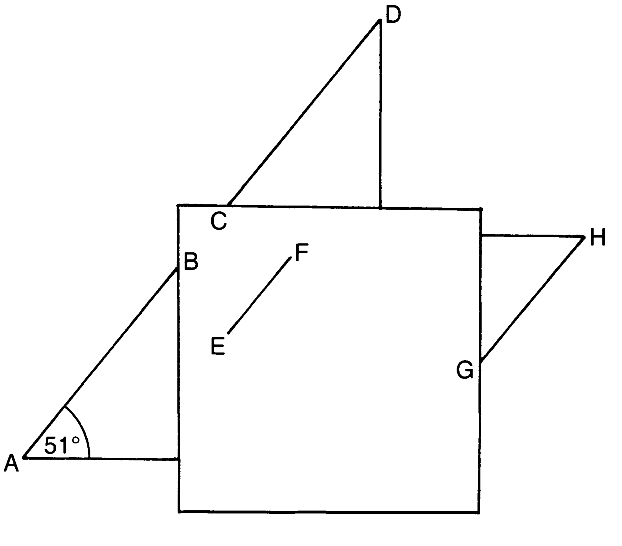

Fig. 5 The styles are all parallel

Note the styles (the edge of the gnomon) AB, CD, EF and GH are all parallel, no matter how the dial block is oriented. For them to be parallel to the Earth's axis the angle between AB and the horizontal must be equal to the latitude of the place where it is used. To do this the appropriate incline can be cut along when constructing the model, or the dial can be put on an inclined plane. For example, at Elgin (latitude 57° 30′ North) the dial will need to be inclined at 6.5° to put the gnomons at the correct angle. To do this put the dial on a 6.5° slope.

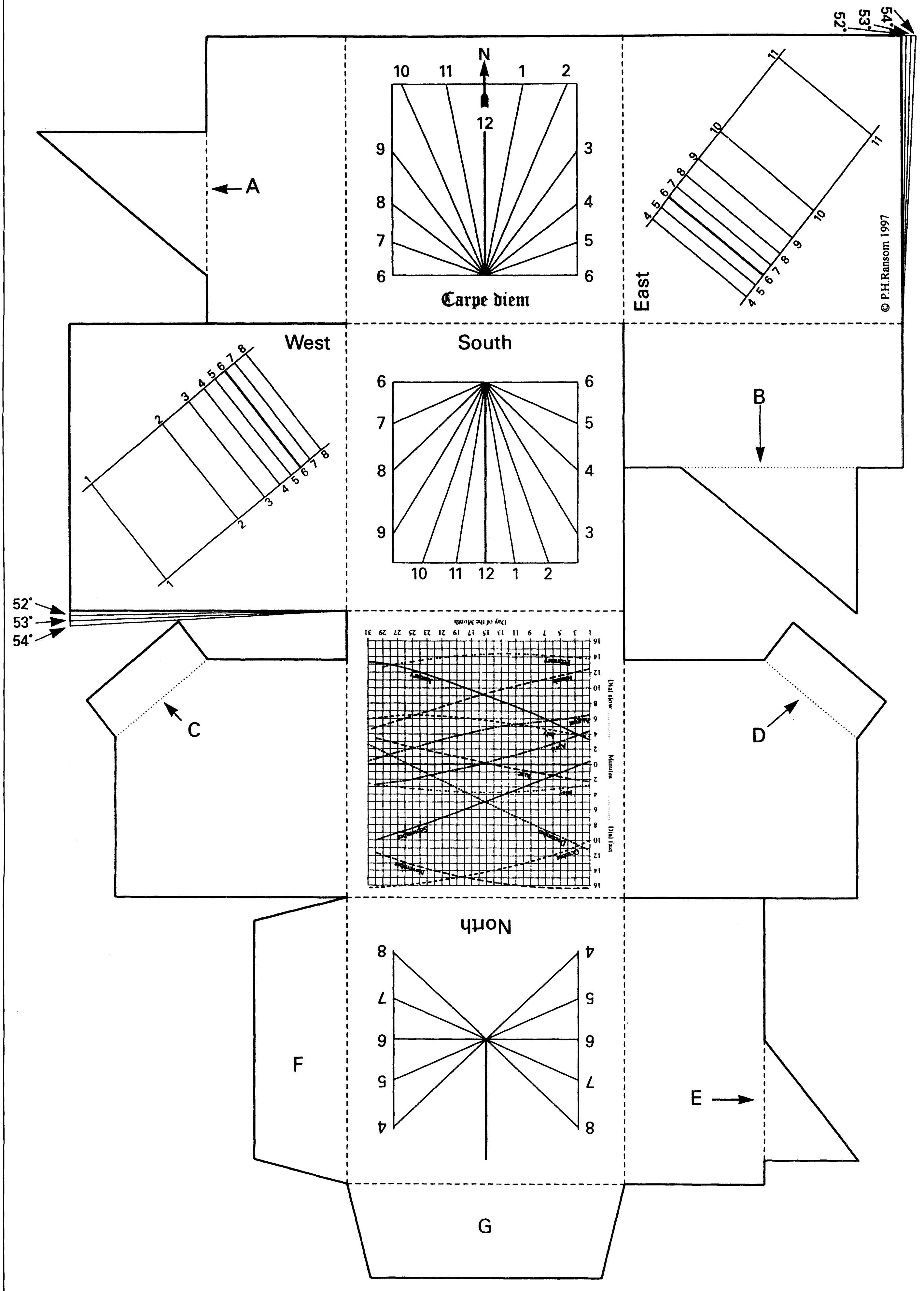

Fig. 6 Net for the construction of the Multiple Dial Block by Peter Ransom

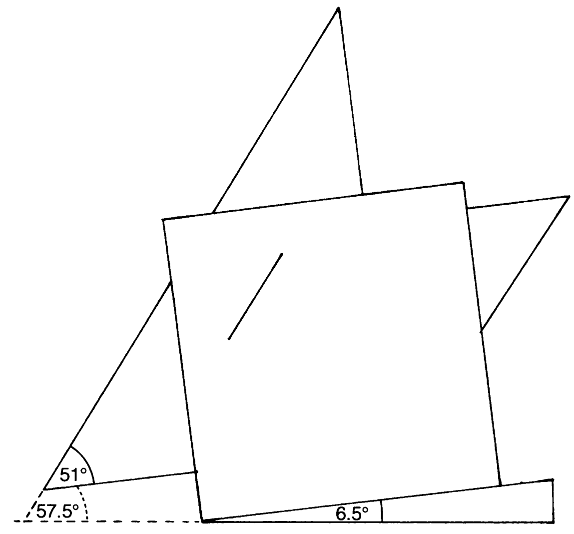

Fig. 7 The dial block on a slope for Elgin

In fact a couple of degrees difference in latitude makes little difference to the accuracy of the time shown. This allows the dial to be erected at the correct angle, but how do we ensure the correct orientation? Since the gnomons must point to the north celestial pole, using a magnetic compass is not appropriate (this gives the north magnetic pole). However, since the dial will not give the correct time unless it is oriented correctly, all we need to do is to turn it until two faces (top and south say) show the same time.

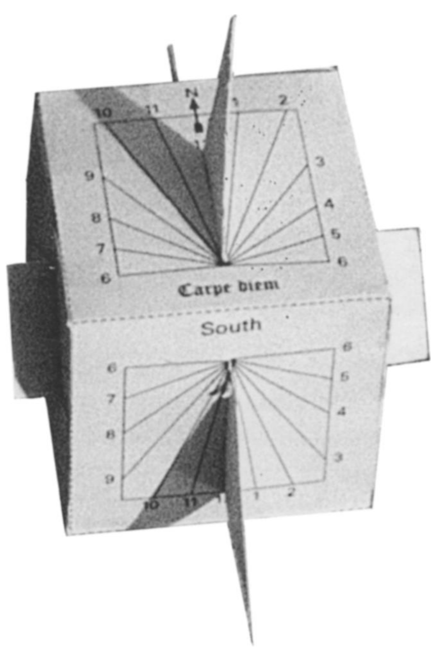

Fig. 8 Dial shown in roughly correct orientation—same time shown on two faces

Another method of finding true north/south, being a practical application of bisecting an angle using ruler and compasses only, is shown in Ransom (1993).

A Bit of History

The principle of the sundial was known to the Chinese as early as 2500 BC, and sundials were widely used by the Greeks and Romans. In AD 606 the Pope is said to have ordered that sundials be placed on churches and this was probably the beginning of the long association of churches with sundials and clocks. The Saxon dial on Escomb church (Co. Durham) is thought to be the oldest dial *in situ* in the UK, dating from the seventh century AD, though the Saxon dial on Bewcastle Cross also claims this honour! Some larger examples, containing information that allow us to date them more accurately, can be found in Yorkshire and Hampshire.

Sundials rely on the casting by the sun of a shadow, of a simple rod or other structure called the *gnomon* (which projects from the surface), onto a calibrated background called the *dial plate*. It is the leading edge of the shadow, cast by the *style* (the name given to the part of the gnomon that casts the edge of the shadow) that is read from the dial plate to find the time. The angle of the style to the horizontal must be the same as the latitude in which the dial is set up.

The measurement of time is based on the rotation of the earth. The interval between successive (apparent) crossings of the sun across the imaginary line drawn through the north and south poles and that place (called the *meridian*) is known as the *apparent solar day*. This is because the shadow of the style on the dial plate depends on the position of the sun as it appears in the sky. Since the earth's orbit around the sun is an ellipse rather than a circle, and its axis is not perpendicular to the orbit's plane, the apparent solar day varies in length up to 31 minutes at the extremes of any year. Since this is not practicable for time keeping by clocks and watches, we average out these variations to produce *mean time* as in Greenwich Mean Time. This means that a correction factor needs to be applied to apparent solar time, which is that measured by a sundial. It was the introduction of the railways, and their need for a standard time for the whole country, that led to the introduction of *Greenwich Mean Time* in 1880.

To adjust a sundial's time to G.M.T. then, we have to apply the appropriate correction (called the *equation of time*) which is obtained from a table, graph, or other chart; adjust for the longitude (add 4 minutes for each degree of longitude west of the Greenwich meridian: for every degree east of Greenwich subtract 4 minutes); and during the summer remember to add one hour for British Summer Time!

For more information about time and the calendar, see Jack Oliver's article on page 2 of this issue.

Practical Problem Solving

Once pupils have made their multiple dials and dealt with the theory (covered in more detail in Ransom, 1993), it is time for some group work. They are asked to bring a clear plastic bottle and wire coat hanger to the session. At The Mathematical Association's Conference I brought some suitable bottles scavenged from the recycling bins, and coat hangers kindly provided by Bollom Cleaners. Most plastic bottles have their centres marked on the bottle top and base. To make suitable holes, cut off a piece of wire coat hanger, heat it in a blow torch flame and push it through the top, then bottom. At the masterclasses I do this while a helper cuts the hook off the hanger and the pupils are making the multiple dial. Instructions for making the bottle dial are given here.

1. Measure the circumference of the bottle.
2. Mark out a piece of paper as shown below, with equal spacing between the hour lines. It is advisable not to make the width less than 10 cm, though it depends on the size of the bottle.

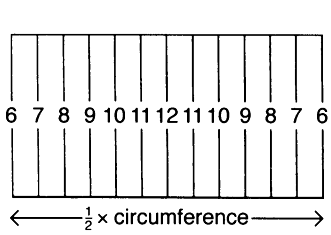

Fig. 9 Dial plate for bottle

3. Make a hole in the bottom of the bottle (and in the top if necessary) with a hot piece of wire the same diameter as the wire coat hanger.
4. Stick this piece of paper inside or outside the bottle as shown in the photograph. If the bottle is not transparent, then you will have to cut out a panel of a suitable size.
5. Straighten out the coat hanger and insert it through the holes at the top and bottom of the bottle. The piece inside the bottle is now called the *gnomon.*
6. Bend the wire so that the gnomon is inclined at the angle of latitude to the horizontal.
7. Place the sundial so that the gnomon lies in the north—south direction as shown. The shadow of the gnomon will fall on the hour lines to show the local solar time.

This is the time to go round offering encouragement and only give suggestions if pupils are truly stuck. It is interesting to watch them try to find the circumference of the bottle. In general an A4 piece of paper will not go around the circumference, and no one has any string! However, by folding the paper along a diagonal allows this to be wrapped around to find the circumference, and progress is made. Some pupils have problems getting the sundial to balance and use a bit of corrugated cardboard to make a suitable base. Others cut an appropriate angle from card to test for the correct angle of inclination. Once these are made it is good to go outside and test them (provided the sun is shining!), but you do need to have done a bit of the following work on reading a sundial.

Reading a Sundial

This section can be used in a classroom without having made any sundials, but is rather sterile without the motivation of the construction of the multiple dial or bottle dial.

It is not often that the time shown on a sundial corresponds with the time shown on a watch or clock. Some basic mathematics is needed to read a sundial correctly, and how this is performed is now described. Here is C. Hunter's mural dial at Hurworth (Longitude 1° 32′ West), England. Hunter was a pupil of William Emerson, a mathematician from County Durham who lived 1701–1782.

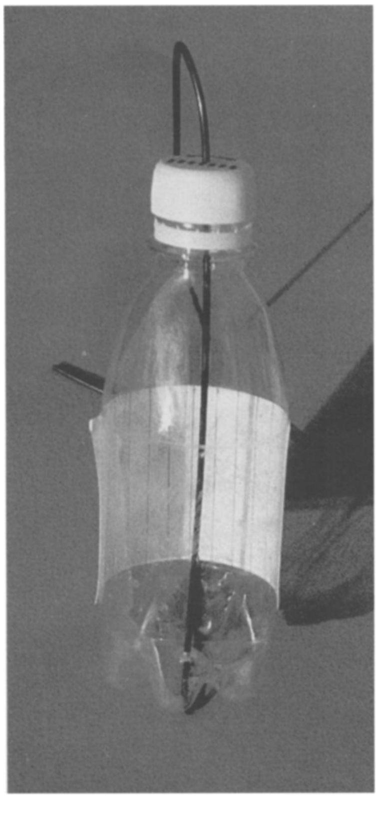

Fig. 10 Dial made from small bottle, showing 2.45 p.m.

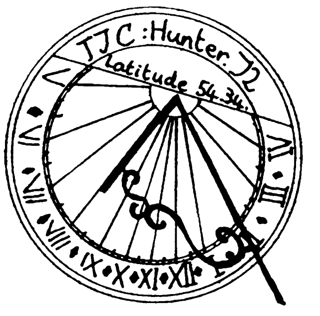

Fig. 11 Hunter's dial, Hurworth, Co. Durham

Here I estimate the time shown by the shadow to be 2.10 p.m.

Next, we need to see if the dial is fast or slow, and make the necessary adjustment. Since the earth does not travel at a constant speed around the sun, it means that the sun's apparent motion around the earth is at times fast and other times slow. Our clocks and watches average out this difference over the year: we use 'mean time'. To adjust for this we use 'the equation of time'. This is not an equation as we think of it, but a table or graph showing how fast or slow the dial is, thus equating the solar time to mean time. It is shown in the form of a multiple graph here.

We now have to do some calculations to adjust for the longitude of the sundial's position. Since the sun appears to go round the earth in an east to west direction, this motion corresponds to travelling through 1° in 24 × 60/360 minutes; i.e. 1° in 4 minutes. Therefore for every degree west of the Greenwich meridian the sun time appears to be 4 minutes behind the mean time.

With these adjustments in mind we can now find Greenwich Mean Time for Hunter's dial shown on 5th August.

- Shadow time shows approximately **2.10** p.m.
- From the equation of time, dial is 6 minutes slow.
  Therefore add 6 minutes: **2.16** p.m.
- Longitude 1° 32′ West corresponds to (1 + 32/60) × 4 = 6 minutes behind G.M.T.
  Therefore add 6 minutes: **2.22** p.m.
- 5 August occurs during British Summer Time!
  Hence we need to add one hour: **3.22** p.m.

Summary

There is a lot of mathematics that can be motivated through sundials. I have described just two activities but higher

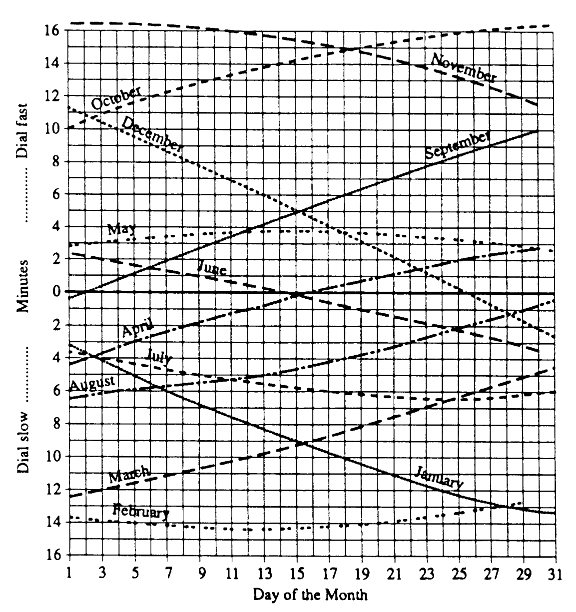

Fig. 12 The equation of time

attainers can explore some of the trigonometry behind making horizontal sundials, or construct sundials through ruler and compass construction. For pupils who are interested there is ample scope for this to form GCSE coursework, and it could be integrated with technology if the dial was constructed from appropriate materials. There is an admirable section in The Mathematical Association's report *The Teaching of Trigonometry in Schools* (1950) on pages 54 to 57 which looks at the subject. Nothing new under the sun!

References

British Sundial Society 1991 *Make a Sundial*, BSS 0 9518404 0 1

Daniel, C. St.J. 1986 *Sundials*, Shire Publications 0 85263 8086

The Mathematical Association 1950, *The Teaching of Trigonometry in Schools*

Ransom, P. 1993 'Astrolabes, Cross Staffs and Dials', *Mathematics in School*, **22**, 4.

Ransom, P. 1998 *A Dozen Dials*, Ransom Southampton.

Waugh, A.E. 1973 *Sundials: Their Theory and Construction*, Dover 0486 22947 5

A Dozen Dials, was published by the author earlier this year and copies are available for £7 (includes p & p, cash or cheque payable to P.H. Ransom,) from him at 29, Rufus Close, Rownhams, Southampton SO16 8LR. A ten page handout of more sundial related activities (some described above) together with 4 sheets to help with the theory and suitable for copying onto OHP transparencies is available from the same address for £2 (includes p & p). *Make a Sundial* is available from Mrs J. Walker, 1 Old School Lane, West Lydford, Somerton, Somerset TA11 7JP for £6 (to UK addresses, which includes p & p). Cheques payable to British Sundial Society.

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Calendrical Calculations

Nachum Dershowitz & Edward M. Reingold
Cambridge University Press 1997
xxi + 307 pages
ISBN 0 521 56413 1 (hb), 0 521 56474 3 (pb)
£40 & £14.95 (US $64.95 & $22.95).

What a wizard wheeze! A set of computer programmes to calculate any date in any calendar and convert between them. Praise, first, for Dershowitz and Reingold, computational calendricologists, who have devoted a large part of their joint lives to a task which anyone in their right mind will be glad someone else did.

Calendrical Calculations is presented by its publishers as "definitive", "accurate", "useful", "easy" and "a must" which, coming from CUP, immediately arouses interest. Its purpose is "to present, in a unified, completely algorithmic form, a description of fourteen calendars and how they relate to one another". The world's main calendars are all here: Christian (both Gregorian and Julian), Hebrew, Hindu (both old and modern), Islamic, modern Persian, Coptic, Mayan and Chinese. There are also three modern reformed calendars, all of them effectively defunct: the Baha'i calendar, the French revolutionary calendar, and the ISO (International Standards Organization) calendar, an excessively sensible Swedish invention. Brief explanations are given of each, and there are valuable overview chapters on calendars in general and on time and astronomy. The bulk of the book, however, is given over to an explication of the algorithms into which the calendars are translated, in a computer language called LISP. These are set out in an appendix. The book comes complete with a licence (yes, you are allowed use it) and an associated website, bristling with errata.

As the millennium approaches, books purporting to explain the calendar are appearing like cactus flowers after a storm, full of second-hand errors, third-order simplifications and outright myths. Dershowitz and Reingold, by contrast, have worked at source and confronted every difficulty. Their book can be recommended as a pithy and reliable distillation of all the world's main calendars. As a bare work of reference, it leads the market. Its corresponding weakness is the need to fix upon one version of each calendar as definitive, whereas all major calendars have in fact been modified and adjusted over the centuries. This robs it of historical value, and makes long-range projections and comparisons unreliable.

The authors indeed point out (p.29) that their method produces answers which are "mathematically sensible, but culturally wrong". Some examples will illustrate this point. The Julian and Gregorian calendars are treated as two distinct entities, even though the Gregorian was in fact a minor correction to the Julian. Its adoption in different states at various stages over the four centuries since 1582 is not tracked; we are offered instead two timeless and unreal paradigms, hypothesizing Easters which never in fact existed. The computer cannot cope with Gregorian countries which observe a Julian Easter (such as Greece), or an astronomical Easter (as with some eighteenth century Protestant states). It is thrown by the Julian hop from AD 1 to 1 BC, and invents a Gregorian year 0, giving out-of-synch BC dates for the two versions of the Christian calendar. The Islamic calendar given is the civil version only, whereas the Islamic calendar is religious; its holy days are determined by observation and announced annually by the religious authorities; they cannot simply be extracted from the civil framework. The existence of a theological divide in the Islamic world between those who measure time by local observation of the new moon and those who accept pips from Mecca is not recognized.

Oblivious to such incalculables, the computer races serenely on, generating absurdities such as Mayan equivalents for 39 December and the Gregorian Easter for the year zero. Curiously, it follows American cultural convention in recording dates in the mathematically illogical form of month-day-year. Finally, there is the little problem of the time of the day. The computer has to cope with different conventions of starting the day at sunrise, midnight, noon and sunset, with local time (general until the mid-nineteeth century), with different time zones, with daylight saving time (which is explicitly ignored) and with events such as Easter, Passover and lunar months which rely on exact observation of the phases of the moon and which can differ by a month depending upon how and where the measurement is done. The solution is to take the day as beginning at midnight but make conversions at noon, Julian time. So, we can give or take a day throughout.

Such problems are generated by the very nature of the enterprise, and Dershowitz and Reingold are well aware of the limitations of the digital approach to calendars. They readily admit (p.28) that "the astronomical code we use is not the best available, but it works quite well in practice, especially for dates around the present time, around which it is approximately centred. More precise code would be time-consuming and complex and would not necessarily result in more accurate calendars." No matter, for the computer programme on which the book is based will soon be as obsolete as the punch card. Here and there, there are hints that the authors (understandably) favour simplified calendars such as the French revolutionary and ISO calendars, from whose short and troubled histories there are surely lessons to be drawn.

The calendar, any calendar, is by its very nature an analogue device, designed to track the incommensurable movements of the earth, moon and sun, to accommodate feasts and holy days governed by arbitrary human rules, and to reconcile conflicts with reference variously to civil, theological or astronomical criteria. No formula can express all that. It is precisely because calendars cannot attain regularity that civil and religious conventions have evolved to govern them. To attempt to reduce these to digital uniformity is sheer hubris. Computers can *mimic* the calendar, just as they can mimic thought, but a computer program will not *be* the calendar, and cannot be interrogated as if it were; we are talking to the monkey, not the organ-grinder. At best, matching calendars is as delicate as mating pandas. At worst, it is as vain as trying to adapt Australian railway trains to run on tramways in Manchester, or trying to find the date of the world cup in pre-conquest America. The history of western attempts, since the enlightenment, to reduce the complex cycles of the human and natural calendars to astronomical or digital perfection is in itself an episode in the history of science whose history, perhaps fortunately, remains to be written.

We can be grateful that so useful a work of reference has been created from a project of such awe-inspiring futility.

Robert Poole

Starting From Maps and Plans

Diane Cobden with Fran Mosley
BEAM 1998
ISBN 18 74099 60 X
50 pages
£6.50 + £2.00 p & p.

In the profession's headlong rush into a tightly structured numeracy-based mathematics curriculum it must be hoped that teachers will not forget the importance of making valid links with other subjects. Such thinking places mathematics in a context and encourages children to get to grips with the meaning of the concept as well as the tricks and skills involved. Indeed the Numeracy Task force did not forget this and highlighted the need for sensible links to be made. Two subjects with clear links are mathematics and geography. Connections have already been made in many of the more recent major schemes, but this book sets out to pull some of the ideas together and suggest the use of geography as a medium for the teaching of mathematics.

It includes open-ended ideas for pupils at Key Stages 1 and 2 exploring plans and elevations, projection, scale and enlargement, map-making, grid referencing, longitude and latitude and the mathematics of globes. The ideas are sound, although occasionally implying the tiresome collection of resources, wrapped up in BEAM's usual high quality production package. Recommended.

Paul Smail