March 2025

YBMA News

Vol. 34 No. 2

The Newsletter of the Yorkshire Branch of the Mathematical Association

Readers may recall receiving a Christmas greetings email proudly listing highlights of the Mathematical Association's year 2024. These included the number of students taking the challenges, the number of new books published and sold, the webinars, the podcasts and the two conferences. Alas, branch activities did not get a mention. However, the February issue of the members' magazine Mathematical Angles went some way towards remedying this omission. Here we find Cindy Hamill's report on the work of the Branches Committee. Cindy is chair of the committee and an active member of the YBMA.

Her article is closely followed by detailed accounts of the two meetings organised by the YBMA in the autumn term. Charlie Stripp, the incumbent President of the MA, came to Leeds in early October to give an in-person-talk entitled "*Key challenges in maths education and how we might address them (and some nice bits of maths!)*". Readers may feel they are already thoroughly familiar with Charlie Stripp's views on maths education in England through his reports and interviews available at https://www.m-a.org.uk/meet-the-president but will have missed the "nice bits of maths" that for those present were perhaps the most memorable part.

In December we held our annual Christmas Quiz, for many of us a not-to-be missed event. A selection of the quiz questions appears in the Mathematical Angles article. If you don't receive the magazine, I would be happy to share these reports with you. Of course, the magazine contains much more. Its 36 pages are full of interesting mathematics, whether just for your own amusement or as classroom material. So better still, join the MA and you will get your own copy through the post.

Like myself, many branch members may not have been aware that our own Tom Roper took over as the MA's Librarian in the autumn of 2024. Tom has been active in maths education not only locally but also nationally and was President of the MA from 2017 to 2018. He has now given himself the task of raising the profile of the MA library, a valuable but much underused resource. Read his article in the February issue of Mathematical Angles.

YBMA Officers 2024-25

President: Lindsey Sharp (lindseyelizab50@hotmail.com)

Secretary & Newsletter: Bill Bardelang (rgb43@gmx.com)

Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

W. P. Milne Lecture

Wednesday, 2 April 2025
14:30 – 15:30

Esther Simpson Building (LG.08)
University of Leeds

Katie Steckles

The Mathematics of Paper

The humble sheet of paper has almost infinite mathematical potential. Join mathematician Katie Steckles as she demonstrates some of her favourite mathematical concepts and shares some puzzles using both real and imaginary pieces of paper. Materials will be provided to join in from your seat, as Katie reveals the mathematical secrets hiding in household stationery.

Katie Steckles is a mathematician based in Manchester, who gives talks and workshops and writes about mathematics. She finished her PhD in 2011 and since then has talked about maths at universities, schools events, festivals, on BBC radio and TV, in books and on the internet.

The lecture forms part of a KS5 Maths Day at the University of Leeds. Applications for school groups have closed, but YBMA members, both current and prospective, can be accommodated. ***Please let us know if you intend to come****.*

A Date for your Diary

Saturday, 7 June 2025
2.30pm

YBMA Annual General Meeting

To be followed by a number of short presentations by members.

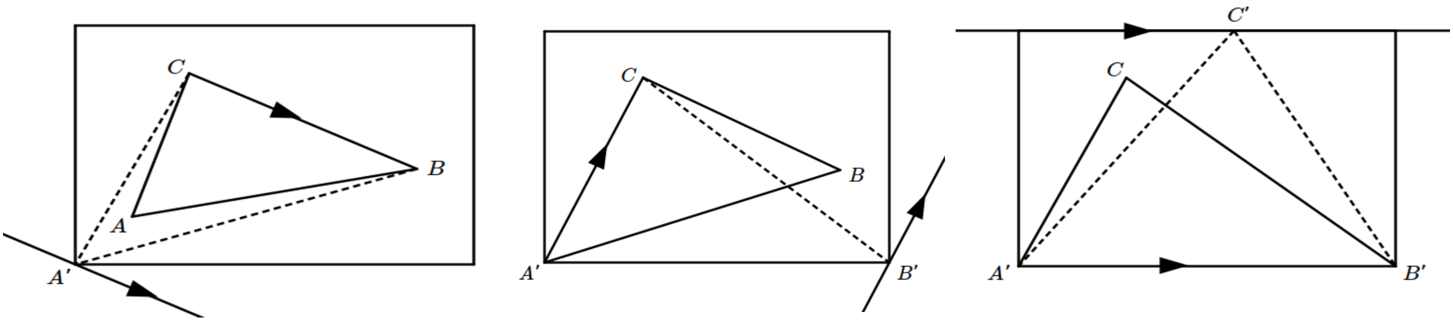
MALL 1, School of Mathematics
University of Leeds

Previous Newsletters can be found at https://www.m-a.org.uk/branches/yorkshire

Mathematics in the Classroom

Paper Triangles

It is intuitively obvious that the vertices of the largest triangle that can be cut from a rectangular sheet of paper will lie on the edges of the sheet. We can illustrate with a sequence of diagrams that two vertices must in fact be at adjacent corners of the sheet and the third on the edge opposite.



Consider each side of the triangle in turn as the base and move the opposite vertex so as to maximize the height. The greatest possible area of a triangle is half the area of the rectangle.

The maximum-sized triangle is clearly not unique. Whatever the dimensions of the sheet of paper it is always possible to have two distinct non-congruent isosceles triangles of maximum area. (Even in the case of a square sheet!) However, if we require the triangle to be equilateral we will usually have to settle for less than half the area of the sheet.

a) For what ratio of length to width of the rectangular sheet is it possible to cut out an equilateral triangle half the area of the rectangle?

b) What fraction of the area is the maximum possible in other cases? Show clearly how the rectangle is to be cut. Can you find the lines of cut by (i) ruler and compass construction, (ii) by folding?

Number Sequences and Recurring Decimals - Solution

In the September Newsletter we looked at the digits of a decimal as a sequence of numbers. Although decimal digits can only take integer values from 0 to 9, we need not impose this restriction on the terms of our sequence.

We gave as an example the sequence of odd integers

1, 3, 5, 7, 9, 11, 13, ...

with its associated decimal 0.(1)(3)(5)(7)(9)(11)(13) ... . Rewritten in 'standard' decimal form, as illustrated on the right, this becomes 0.1358024… . We claimed this to be the decimal equivalent of the rational number $\frac{11}{81}$ with period 9.

```
0.1 3 5 7 9
          1 1
            1 3
              1 5
+               ...
_________________________
0.1 3 5 8 0 2 4 ...
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The reader was asked to investigate the decimals associated with two well-known sequences.

a) The sequence of square numbers 1, 4, 9, 16, 25, … .

Our starting point is the sum of the infinite geometric series $1+t+t^2+t^3+t^4+\ldots=\frac{1}{1-t}$.

Differentiating w.r.t. t , then multiplying by t and then the same again, we arrive at

$$t+4t^2+9t^3+16t^4+25t^5+\ldots=\frac{t(1+t)}{(1-t)^3}\quad.$$

Substituting $t=\frac{1}{10}$ gives 0.(1)(4)(9)(16)(25)… = $\frac{110}{729}$.

b) The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, … .

From the third term onwards, each term is the sum of the previous two, thus if

$$\begin{aligned} N &= t+t^2+2t^3+3t^4+5t^5+8t^6+\ldots \\ tN &= \quad t^2+t^3+2t^4+3t^5+5t^6+\ldots \\ t^2N &= \quad\quad t^3+t^4+2t^5+3t^6+\ldots \\ (1-t-t^2)N &= t \end{aligned}$$

Substituting $t=\frac{1}{10}$ gives N = 0.(1)(1)(2)(3)(5)(8)… = $\frac{10}{89}$.

In both cases, displaying the full recurring sequence of digits of the 'standard' decimal form of N is beyond the scope of a pocket calculator. A web-based calculator, e.g. WolframAlpha could be used.

In the case of division by 89, a prime, there are 88 non-zero remainders and the division process will repeat after 88 divisions. It may repeat after n divisions, where n is a factor of 88. The least value of n such that $10^n\equiv1(mod\,89)$ gives us the length of the recurring sequence.

$$10^1\equiv10(mod\,89)\ ,\ 10^2\equiv11(mod\,89)\ ,\ 10^4\equiv32(mod\,89)\ ,\ 10^8\equiv45(mod\,89)\ ,$$
$$10^{11}\equiv55(mod\,89)\ ,\ 10^{22}\equiv88(mod\,89)\ ,\ 10^{44}\equiv1(mod\,89)\ ,$$

so we can conclude that the decimal equivalent of $\frac{10}{89}$ has period 44.

In the case of division by 729, there are 486 remainders coprime to 729. We need the least value of n such that $10^n\equiv1(mod\,729)$, where n is a factor of 486. The reader may like to check that this value is 81 and the decimal equivalent of $\frac{110}{729}$ has period 81.