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New challenges

The 1989 Presidential address

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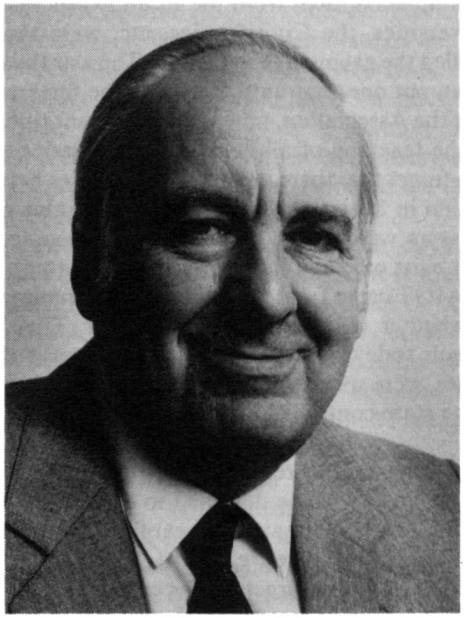

Some eighteen months ago I took advice from a friend on what might be a suitable topic for this Presidential address. "Do something historico-political", he suggested. In the event I have had little option, for in those eighteen months the climate of mathematics education in our country's schools has changed in an unprecedented manner: the mathematics curriculum is now very much a political issue.

George Tomlinson, Minister of Education in the late 1940s, is credited as declaring, "T'Minister knows nowt about t'curriculum". It was, of course, intended as a statement of policy rather than of fact. Nowadays, whatever the fact might be, it is a statement of policy that, so far as the curriculum is concerned, the Secretary of State is omniscient. We are having to adjust to the imposition of a National Curriculum and testing procedures which, in their extent, not only have no parallel in English educational history, but, so far as their structure is concerned, in no other country. Moreover, it is an imposition marked by a frenetic timetable, insufficient thought and consultation, and the setting-aside of much of the advice offered.

The effect on members of the Association is bound to be—indeed has already been—significant. I intend to argue that the effect on the Association itself must also be very marked. Recall that our Association began in 1871 as "The Association for the Improvement of Geometrical Teaching": its prime aim being "to promote the general improvement of geometrical teaching, and . . ., as a necessary preliminary, [to] use all its efforts to induce [examiners] . . . to frame their questions independently of any particular textbook [i.e. Euclid]". The examiners, drawn from the recently established Examination Boards, the Universities, the Civil Service, etc, were those who, in 1871, effectively controlled the geometry curriculum. The war the AIGT fought was to be a long-drawn out one; not until 1903 was the final and decisive battle won. By that time the Association, now with its present title, had extended its aims to include the teaching of all fields of mathematics.

Yet it is easy to forget that there are many similarities between the national educational position in 1871 and that today. For what we must remember is that our predecessors were a very select group of people. State secondary education did not exist officially in the nineteenth century, and the teachers belonging to the AIGT came from the old endowed grammar schools and the public schools, whether old, or newly established. They, like teachers in independent schools today, although constrained by the examining boards and the universities, were spared the circumscription of syllabuses and testing to be found in the state-controlled elementary schools and teacher-training institutions.

The principle of "payment by results" was presented to Parliament in 1862 by Robert Lowe. It was a scheme designed to ensure that the size of grants paid to schools was related "to the attainment of a certain degree of knowledge by the children in the school" (see, e.g. [2]).

So as better to fix attainment targets in the core subjects of reading, writing and arithmetic, certain levels (or standards) were laid down. For example, in 1862 short division was Level 3 (now, in 1989, it is Level 4). Later, other, optional topics were added to this core, such as algebra, geometry, natural philosophy [the physical sciences], history, geography, the natural sciences, political economy, and languages (i.e. English literature or the elements of Latin, French, or German). Drill [physical education] and singing were encouraged [2, p 370].

Attainment targets were regularly adjusted and so by 1871 we find that, for example, Level 1 comprised simple addition and subtraction of numbers of not more than four figures and the multiplication table to multiplication by six.

Today, of course, we have gone decimal and so now Level 3 is to know the multiplication table up to 5×5. Yet, even in 1871 attention was given to the metric system. Schools were told that "the weights and measures taught . . . should be only such as are really useful . . .. In all schools the children in Standards V and VI should know the principles of the Metric system, and be able to explain the advantages to be gained from uniformity in the method of forming multiples and submultiples of the unit" [11, p cix] (so how was it that in junior school I still had to learn about rods, poles and perches?). Alas, parliament, which is not always distinguished for its appreciation of scientific matters, failed to understand what those advantages were; the move towards metrication faltered, and in 1874 this reference was removed from the *Code*. One result was that in the Second International Mathematics Study, carried out in 1981, only 35% of English and Scottish students could say which of 8·5 kg, 85 kg, 185 kg, 850 kg and 1850 kg would be most likely to be nearest the weight of a normal man. Nine European systems had scores of 90% or above on this item. Less obviously, but, I feel, no less certainly connected, was the dismal performance of our schoolchildren on many items related to decimals. Here, then, we have an interesting example of how a political decision taken over a century ago still influences mathematical "attainment" in our schools today. A striking example to demonstrate that not all student shortcomings can be attributed to the teacher.

The standard assessment tasks were in those days constructed and administered by Her Majesty's Inspectors and so it is interesting to look at their reports concerning the effect of these measures on schools. I have not gone to Matthew Arnold, the traditional source of quotations on this theme, but felt that, since we are in Sheffield, I would turn to the 1871 Reports of two HMIs concerned with inspecting Yorkshire schools. I should like to emphasise that these were the *only* two reports which I read; I have not picked and chosen to reinforce my own point of view.

Mr French [11, pp 72–81] reported that "the teachers are so anxious that their children should pass . . . satisfactorily that they screw the poor little bairns up to the very point of being just able to do the required sums, and nothing beyond. . . . I wish teachers would remember that what is required for a pass is the minimum and not the maximum amount of knowledge that the children are expected to possess; and that . . . to pass a really satisfactory examination, they should be instructed from the earliest period, beyond what is absolutely required". Surely, Mr French was not suggesting that "Programmes of Study" should comprise more than the "Attainment Targets" to be assessed at that level!

Mr Wilde (pp 238ff) had worries about another issue: that of publishing league tables of schools based on the results of examinations: "In judging the

efficiency of a school, one must be guided by the school itself and weigh well its peculiar circumstances, as these vary so much. For this reason I should be very sorry indeed to have to publish a list of my best schools. A school may be very good under its own peculiar circumstances, and yet when compared in any definite particulars with others might be found sadly wanting . . . . I have several schools where from the results of the examination I am obliged to give what may be considered a flourishing report, whereas all the time I am far from satisfied with the tone. On the other hand many a school with far less satisfactory results is really doing a much greater work in education." League tables were, however, published for teacher training institutions and, of course, syllabuses for these were centrally laid down: arithmetic, Euclid and algebra for male student teachers, but only arithmetic for females.

We can, of course, find precedents for other recent initiatives. The 1904 regulations, intended to prevent newly-established state secondary schools developing too scientific and technological biases, laid down minimum hours to be devoted to certain core topics within the curriculum: "not less than $7\frac{1}{2}$ hours to Science and Mathematics, of which at least three must be for Science" and in girls' schools at least one-third of the total number of hours (if less than 22) provided again that 3 hours were devoted to Science (see, e.g. [10, p 156ff]). Perhaps, however, we can draw cheer from the fact that frequent external testing, "standards" (in the sense of 1862), league tables and the 1904 regulations all passed away once their political purposes had been served. History can offer encouragement!

Yet, here it must be stressed that I am not suggesting that changes are not urgently required in English education. A national curriculum setting out which subjects should be studied and the relative emphases to be placed on these has much to offer. One merely regrets that more thought was not given to its construction, to the realities of implementation, and to the needs of the non-academic pupil. In particular, though, serious consideration must be given to many aspects of current mathematics teaching. Neither the *Cockcroft Report* nor the latest proposals of the National Curriculum Council give sufficient indication how we can remedy a situation in which too many students underperform and not enough are encouraged to continue with the study of mathematics post-16.

I shall not repeat here the dismal statistics emerging from the Second International Mathematics Study (see, e.g. [4], [9]). But I should like to consider two particular items set to 13 and 14 year-olds.

I find the results of these two items extremely worrying.

First, have we reason to doubt the validity of the 1981 data? In the case of place value the answer is an unequivocal "no". Cockcroft tells us that "not until the age of 15 are at least half the children in a year group able . . . to state that the 1 in the number 2·31 represents 1 hundredth" [3, para 341]. There is some doubt about the item on fractions, for Concepts in Secondary Mathematics and Science (CSMS) and Assessment of Performance Unit

Item 007

847·36

In the number in the box the digit 6 represents

A $6 \times \frac{1}{100}$
B $6 \times \frac{1}{10}$
C 6×1
D 6×10
E 6×100

Country	Percent correct	Percent "taught the topic"	Average age of students (in months)
Belgium (Flemish speaking)	75	83	170
Canada (B.C.)	74	96	168
Japan (2)	72	92	162
Japan (1)	70	—	154
England and Wales	50	93	170

Item 003

$\frac{2}{5} + \frac{3}{8}$ is equal to

A $\frac{5}{13}$
B $\frac{5}{40}$
C $\frac{6}{40}$
D $\frac{16}{15}$
E $\frac{31}{40}$

Country	Percent correct	Percent opting for A	Percent "taught the topic"	Average age of students (in months)
Japan (2)	89	4	100	162
Japan (1)	84	4		154
France	72	12	100	170
England and Wales (1964)	63	19		173
England and Wales (1981)	42	35	97	170

(APU) data suggest that perhaps 50% giving correct responses and 25% opting for "adding top and bottom" might have been expected. However, APU tells how "the lowest two [of five] bands of attainers cannot cope at all with different denominators" [5, p 132], while CSMS writes of a *decline* in

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attainment on such problems as the child gets older [6, p 79]. Such findings would not appear to be consonant with the data from other countries given above.

Now it might be that you believe that the ability to add fractions is not one that is needed by many pupils these days, since it is a skill rarely demanded in "real life". Again, if you believe that other countries are misguided in attempting to teach probability or calculus, with its algebraic demands, to so great a proportion of the age cohort, then you will find such work unnecessary. But it was *not* the case that teachers claimed not to have taught the addition of fractions. We must, therefore, ask the key question: *what does it mean to say that the subject has been taught, if in fact so many students appear to have no grasp of the topic?*

The same question springs to mind time and time again as one examines the SIMS data (and not only those relating to England). The proposed National Curriculum offers no help here. It was *not* the case that we did badly because our students had not been taught topics. Syllabus coverage was no more haphazard in England, than in countries, such as France, with a national curriculum. True, in Japan the coverage of items in arithmetic and algebra was more uniform than in England, but, even in Japan, this was not true of items in statistics and geometry.

Although there might be doubts about the need for all to be able to add $\frac{2}{5}$ to $\frac{3}{8}$, there can surely be no question about the desirability of everyone comprehending place value. But what do we find if we consider Parliamentary Orders? "Place value" has now "drifted out" to Level 6; (i.e. to explain that 0·23 is two tenths and three hundredths, or twenty three hundredths is now Level 6.) That is, we expect that 35% of our 16 year-olds will not understand it, and that the median child will not comprehend it until the age of 15. This is in a world in which calculators are increasingly used. What makes "place value" Level 6 work? Is it that we believe that children cannot deal with it before that age? That is, do we ignore what happens in other countries? Or do we genuinely believe that all the topics proposed to be studied at lower levels are mathematically more important and more accessible?

I find it very disturbing that we should have all been drawn into the parlour game of assigning topics to levels, to attainment targets and to profile components, rather than tackling in a scientific and non-politically-frenetic way the true problems which face us. What is it we value in school mathematics and why; what is likely to be attained rather than merely aspired to? What are reasonable teacher and pupil expectations and how can they be

Gleanings

New unattainable challenges

"Let us take one example [of proposed levels in the National Curriculum], scientific notation, e.g. expressing 22 731 as $2{\cdot}2731 \times 10^4$. This is level 7^{32}." From Geoffrey Howson's booklet *Maths problem*, sent in by Douglas Quadling.

adopted without engaging in chimera hunting? What are the lessons for us if attainment in particular topics does decline with age? What outcomes might a teacher realistically expect from a series of lessons, or from pupils engaged on "individualised" tasks? What does it mean to say that the pupil has "learned" such and such from these activities? How is such "learning" to be assimilated, consolidated, linked, revised and reinforced? These are some of the questions that we should be addressing, not merely how "learning" will be tested.

Readers of my booklet published earlier this year by the Centre for Policy Studies [9] will know of my opposition to the TGAT model: the corset into which the National Curriculum is being forced.

I shall not repeat my arguments today. Suffice to say that I am still astonished and dismayed that such a model should have been accepted and applied across the whole curriculum, not only without adequate time for thought and discussion, but also without any attempt to consider and to exemplify what it meant in detail within any one subject area. Let us hope that my fears are misplaced. If not, then it will require a massive upheaval to dismantle the apparatus currently being constructed at such cost—and a correspondingly massive loss of face. The historical omens are not good. Lowe's reforms were soon heavily criticised and in 1964 he resigned following accusations of having censored the reports of HMI by removing critical statements and opinions. (The independence of HMI and their reports remains, of course, vital.) Yet the apparatus he had constructed, although constantly amended, remained in operation for another thirty years.

There are many problems, then, within mathematics education which demand resolution or at least amelioration. I believe that the government is pinning too much hope, particularly at the secondary school level, on the imposition of uniformity and on additional testing as means of increasing levels of attainment. Benefits would well accrue from the drawing-up of nationally agreed criteria at 11+—criteria which took into account the fact that children differ considerably in ability and attainment—and even from the imposition of some broadly-based assessment of student attainment at that age. However, tests along the proposed lines at 7+ and 14+ would not only seem potentially unproductive but also to absorb far too many resources in an educational system which becomes increasingly under-resourced. (The British Surgeon General is reported to have replied to complaints about the breakdown of the medical services in the Crimea by claiming, "The medical services would have been perfectly adequate if it had not been for the casualties" [13, p 18]. No doubt, the provisions for the introduction of GCSE would have been "perfectly adequate" had it not been for the students, and the promises made relating to INSET and the National Curriculum would, I am certain, be kept if it were not for the teachers.)

The greatest shortage is, of course, that of qualified teachers. This is a long-standing problem to which previous Presidents, for example, Dr Kerr in 1977, have referred. In recent years the demographic dip has served to ease demands and the problem has been hidden because of classes being taken by

unqualified teachers or through timetabling changes. Now the demographic dip is passing through our secondary schools. Research carried out by Professor Smithers and Dr Robinson of Manchester University, supported financially by the Headmasters' Conference, the Secondary Heads Association, and the Engineering Council, has resulted in some horrendous forecasts. "In mathematics, the optimistic supply curve (assuming wastage reducing to 5%, and PGCE and concurrent training output increasing by 30% over three years) shows the shortfall widening to 4141 by 1995. The pessimistic supply curve (wastage rising to 13% and PGCE and concurrent training output reverting to pre-1987 levels) shows a catastrophic situation with a shortfall of 12 232 in 1955 [i.e. 55·7% of demand]" [12, p 20]. In fact the optimistic figure of 30% shows signs of falling considerably this year.

What does one do when faced with a problem of that order? Are the National Curriculum and proposals for testing merely smokescreens to divert attention from major problems? How can we retrieve the situation? For example, although I have doubts about much that passes nowadays for "individualised learning", it may be well that resource-based learning will be all that many state schools will be able to offer in the mid-1990s. It is essential then that intensive research and development work would be undertaken immediately to see how such teaching methods can be improved and their deficiencies diminished. Certainly, it is no use expending all our energy on attempts to solve the teacher shortage problem. It will not be solved. It can only be coped with. The inertia and shortsightedness of a sequence of governments has led to this sorry state of affairs. By all means let us make every effort to increase recruitment and, even more importantly, to decrease wastage and to win back some of those who have left teaching. It is essential that such steps are taken, otherwise the status of teachers and of teaching will sink even lower. But now is the time when the government *and* the Association should be making contingency plans for dealing with the potentially catastrophic situation which schools will experience in five or so years' time.

What of pupils? What can be done to increase their motivation and their confidence in their ability to do mathematics? No major developed country would appear to have so few of its 17 year olds studying mathematics. To some extent this is due to the educational and mathematical opportunities offered post-16. As they stand, A-level and AS-level are not suited for more than a small minority of pupils. Other courses would appear to have little to attract enquiring minds. The more GCSE is successful, the less attractive these post-16 courses will appear. Yet, I do not believe that post-16 take up is governed solely by post-16 opportunities: pre-16 failure and pre-16 labelling must also bear much of the blame. The new form of classification by levels beginning at entry to primary school—or even before—will not help. Syllabuses for pupils currently taking papers 2 and 3 of a four-in-line scheme must be reconsidered. For too long we have tended to think of this work as the last mathematics the pupil is likely to be taught. This is not the case in other countries for pupils drawn from this section of the ability band. Here again I believe the TGAT

model to be unfortunate. Such pupils do not want to be branded as being at, say, Level 6 of an academically-oriented, ten-level course, with all that entails in loss of morale. A learner course geared to the pre-requisites for further mathematical study could be more appropriate for them. We must not cut off such students from, say, careers in engineering by insisting that they study topics which, although meriting a place in the curriculum of high-attainers, cannot be seen as essential pre-requisites for further study. To do so would be in neither the students' nor the country's interest.

Again, are methods of teaching based on frequent changes of topics, particularly when our syllabuses are broader than those in other countries, likely to be successful? One lesson of SIMS appears to be the effect of "intensity" of treatment on student learning. It is not sufficient to touch on something and if nothing is retained take consolation in the fact that the child will be encountering it again in six months' or a year's time. Here, though, we are really faced with a dilemma, for "intensity" of treatment is counter to almost everything the student meets in society. In his *Culture and anarchy*, Matthew Arnold wondered whether it was possible to bring culture to a people whose major reading matter was the *Daily Telegraph* [1, p 59]. What would he have thought if confronted with today's tabloid press? Mathematics can never be part of the "three-minute culture": it demands of those who wish to learn it, concentration and effort over time. Yet these qualities are becoming increasingly alien to our society. The popular newspapers and TV and radio programmes targetted at teenagers assume the concentration span of a backward gnat.

This not only makes our tasks as teachers more difficult, but it undermines a key educational goal, that of extending the student's concentration span. However, as with so much that we value, this quality is not one which is highly regarded by our society.

This, of course, brings us to what is probably the key issue which confronts us. To what extent does society understand and accept the educator's aims, and, in particular, the aims of mathematics teachers and lecturers? Is there, in fact, any consensus amongst us, the teachers and lecturers, on what those aims are? It really is essential for the future well-being both of our subject and of our country, that we as educators begin to express ourselves with greater clarity, unanimity and force. The effects, both potential and now unfortunately actual, of running down an educational system must be made explicit to governments and those who elect them. The status of the teacher was allowed to sink during the late 1970s and early 1980s. Government and teaching unions were locked in a battle which the Government, as paymaster and, to a large extent, controller of the media, was forced to win. But at what cost to education was that battle won? Now the battle is on in the universities and higher education. Deteriorating and dispiriting circumstances have led to serious rifts appearing within the university community: staff have been set against managers in the shape of the Committee of Vice-Chancellors and Principals. Once again, education will be the sufferer. Lecturers or teachers

have again been depicted as putting themselves and their pay packets before their students. Yet which students stand to lose most: those in universities this year or those who will hope to have a university education in the first decade of the next century? The shortage of mathematics teachers in schools in the 1990s is now a certainty: are current policies ensuring that our universities will soon face a similar situation? The Hillgate Group, it is reported, recently argued [7] that school teaching should be opened up to an untrained, non-graduate "businessman or ... part-time mechanic who has the knack of explaining *his* knowledge to children" [my italics]. (I recalled sadly how Thomas Tate in the 1850s looked back in horror to those times when a "tradesman [who] failed in business ... was thought to be learned enough for a schoolmaster" (see [8], pp 101–2). Whom in the 2000s will the Hillgate Group propose as potentially university lecturers?

Once a profession's status has sunk then it is very difficult to raise it again. A recent editorial in the *Daily Telegraph* (March 1, 1989) argued that "to shower all our 400 000 teachers with more money ... would be pointless. Far too many ... would not begin to provide value in return." One can see the logic in this argument. Unfortunately, that this would be the inevitable outcome of policies pursued in the past fifteen years was not foreseen. Education demands long-term planning. Matters can never be set to rights quickly: improvements depend principally upon the competence of the teaching force and the time and resources needed to replenish and retrain it are substantial.

Yet how are we as an Association to respond to these problems? What should our priorities now be? Certainly, I cannot attempt to answer this question on behalf of the Association. It is one, however, that demands immediate and detailed consideration. One possible answer is that we must put all our energies into trying to make the National Curriculum and testing work. Yet this argument makes what appear to me to be two unjustifiable assumptions: (a) that the proposals are workable, and (b) that they would result in a general improvement in the level of mathematics education in this country. The Association's objective is, of course, "to effect improvements in the teaching of mathematics and its applications", *not* "to implement governmental policies as effectively as possible". Yet here it is essential to point out that we, as an Association, now have to perform two very different duties. One, is to fight at a national level for improvements in mathematics education, in a manner which is essentially independent of the government policies of the day. The other is to give support to the teacher in the classroom who is having to cope with such policies. We cannot afford to ignore either of these objectives.

It could be argued that to a large extent we have contributed unwittingly to the situation in which we now find ourselves: that we did not respond to obvious trends. Perhaps, I might be allowed to quote the closing paragraph from my *History of mathematics education in England*, published in 1982: "The arrangements proposed by Sir Keith Joseph [i.e. the establishment of the SEC

and SCDC] would appear to presage further moves towards 'central' control of the curriculum. In the first draft of my *Postlude* [i.e. chapter on history post-1960] I saw as 'the most significant event' of the last two decades, the way in which the Department of Education and Science was seeking to exercise greater control over the school curriculum. Friends, . . ., persuaded me to rewrite the passage on the grounds that I read too much into what were chance decisions. Recent moves have served to reinforce my earlier apprehensions. This is not to say that greater central control of the curriculum is necessarily wrong. What it does mean, however, is that educators whould be aware of any drift and of the probable consequences of that movement and should frame appropriate responses. A move towards a centralised curriculum would indeed be the most significant of contemporary trends in English mathematical education" ([8], p 280).

In the event, the attention of mathematics educators at school level became largely pre-occupied with attempts to implement the *Cockcroft Report*. The political significance of comparative studies such as those from the National Institute for Economic and Social Research and SIMS went unheeded, and the opportunity to take the initative by making curricular proposals consonant with the readily observable political drift was lost. As a result we must now spend much time in reacting to a variety of imposed "solutions" to our more publicly-discernible problems.

It is essential, then, that in future the body of mathematics teachers and educators should think far more about policy at a national level. We must attempt to lead and to influence, rather than to react to *faits-accomplis*. Again, though, the problems inherent in such a statement must be recognised. First it must be admitted that the mathematics community is at present a much divided one. We cannot expect our message to be clearly heard when it is presented as a piece for several solo voices, not always in harmony. At last, serious efforts are being made to bring together the various associations which represent our interests. Such work must be pursued with vigour, but collaborative policy-making, based on joint analyses of data and problems, would seem to me to be more deserving of our time and energy in the short term than attempts to overcome the administrative and other attendant problems of amalgamation. Balancing these two demands on our resources will not be easy.

More important though is the need to develop channels of communication with those who make policy-decisions. What is the use of framing policies, of making proposals, if one cannot effectively communicate these to the decision-makers; if one cannot make them aware of the true context in which their decisions are being made? Our Association, the Royal Society and other bodies have spent considerable time over the past twenty years trying to persuade DES of the critical position concerning the supply of qualified mathematics and science teachers. Alas, the messages fell on deaf ears. Now that the ES has added the National Curriculum to its concerns, is it likely that they will have more time to spend in listening and learning, or will "acting"

now completely fill their days? We must hope not, for it is essential that the voice of professionals should be heard. If it is not, then our claim to be professionals will cease—the Association's motto, "I hold everyman a debtor to his profession", will have become outdated.

Let us hope that this does not come to pass.

My address may have seemed a gloomy one. It would have been unrealistic to have made it otherwise. Nevertheless, I hope I have indicated ways in which we should be thinking and moving, and have helped identify important issues which both the Association and the country must face. For us, a new key question now is "How is the Association to be most effective in the context of new government policies?", for society, and the Parliament it has elected, it must be "How do we value education, and those whose profession it is to educate?".

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A colourful path

TAT-SANG FUNG

(This article is the result of some work done by a first-year undergraduate, Tat-Sang Fung, at the University of Hong Kong. It went way beyond what his lecturer, Man-Keung Siu, had asked for, and he was so impressed that he submitted it to the *Gazette*. I have edited it a little, but the ideas are entirely Tat-Sang's.)